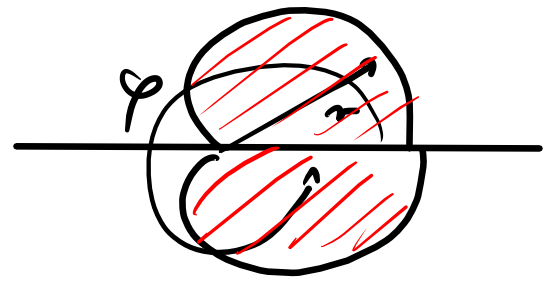
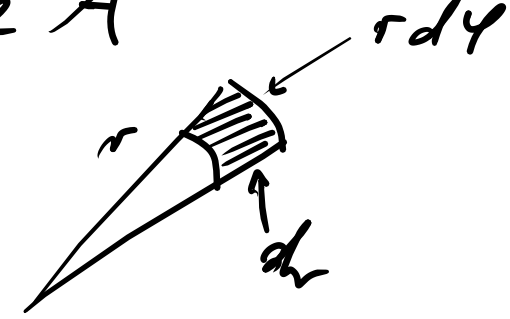
Tutorium BMC M 2/3 22.10.26

Kardioide



gesucht Fläche A

$r(\varphi) = 1 + \cos\varphi$ $\quad 0 \le \varphi \le 2\pi$



$$\int_0^{2\pi} d\varphi \int_0^{1+\cos\varphi} r\,dr = \frac{1}{2}\int_0^{2\pi} d\varphi \cdot \left[\frac{1}{2}r^2\right]_0^{1+\cos\varphi}$$

$$A = \int r\,dr\,d\varphi = \frac{1}{2}\int_0^{2\pi} d\varphi \cdot (1+\cos\varphi)^2 - 0$$

$$= \frac{1}{2}\cdot\int_0^{2\pi}(1+\cos\varphi)^2\,d\varphi \xrightarrow[\text{Tabelle}]{} \frac{1}{2}\int_0^{2\pi}\left(1 + 2\cos\varphi + \cos^2\varphi\right)d\varphi \Longrightarrow$$

$$= \frac{1}{2} \int_0^{2\pi} \left(1 + 2\cos\varphi + \frac{1}{2} + \frac{1}{2}\cos(2\varphi) \right) d\varphi$$

$$= \frac{1}{2} \left[\frac{3}{2}\varphi + 2\sin\varphi + \frac{1}{4} \cdot \sin(2\varphi) \right]_0^{2\pi} = \frac{1}{2} \cdot \left[\frac{3}{2}\varphi \Big|_0^{2\pi} + 2 \cdot \sin\varphi \Big|_0^{2\pi} + \frac{1}{4} \sin(2\varphi) \Big|_0^{2\pi} \right]$$

$$= \frac{1}{2} \cdot \left[\frac{3}{2} \cdot 2\pi \right] = \frac{3}{2}\pi \approx 4{,}7124 \text{ F.E.}$$

Lösen Sie die DGL $y' = \frac{x}{y} + \frac{y}{x}$ mit Substitution

$$\boxed{\frac{y}{x} = u}$$

$y = x \cdot u(x) \Rightarrow y' = u + x \cdot u'$

$u + x \cdot u' = \frac{1}{u} + u \Rightarrow \int u \, du = \int \frac{1}{x} \, dx$ T.d.V.

$\frac{1}{2} u^2 = \ln x \rightarrow$ rücksubst.

$\frac{1}{2}\left(\frac{y^2}{x^2}\right) = \ln x \Rightarrow y = \pm x \sqrt{2 \cdot \ln x}$

DGL zu lösen: $y' + \frac{1}{x} \cdot y = \frac{x^2+1}{x}$ Variation der Konstanten

$y' + \left(\frac{1}{x}\right) y = x + \frac{1}{x}$

$a(x) = \frac{1}{x} \quad g(x) = x + \frac{1}{x}$

$A(x) = \int a(x) = \ln(x)$

$$\Rightarrow y(x) = \left(K + \int e^{\ln(x)} \cdot \left(x + \frac{1}{x} \right) dx \right) \cdot e^{-\ln(x)} \quad \text{(Formel)}$$

sei $y(3) = 0$

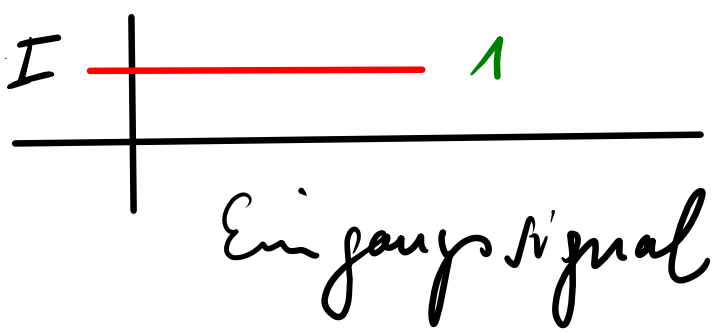
$$= \left(K + \int (x^2 + 1)\, dx \right) \frac{1}{x} = \left(K + \left(\frac{1}{3}x^3 + x \right) \right) \frac{1}{x}$$

$$= \frac{K}{3} + \frac{1}{3}x^2 + 1 \quad \text{mit} \quad y(3) = 0 \Rightarrow \underline{\underline{K = -12}}$$

Zu lösen mit Laplace:

$$y' + 2y = 2t - 4 \quad \text{m. } y(0) = 1 \Rightarrow$$

$\mathcal{L}$.$\mathcal{T}$.: $Y(s) - 1 + 2Y(s) = \frac{2}{s^2} - \frac{4}{s}$ $\Rightarrow$ $Y(s)$ auflösen



auf Rondelator $\Rightarrow$

