Tutorium Math. 2/3 BMC 19.11.26

erst mal was Leichtes:

$\iint$ mit $3 \geq x \geq 1$; $\frac{\pi \cdot x}{2} \geq y \geq 0$ über $\cos\left(\frac{y}{x}\right)$

$$\Rightarrow \int_{x=1}^{3} \int_{y=0}^{\frac{\pi x}{2}} \cos\left(\frac{y}{x}\right) dy\, dx \Rightarrow \underline{\underline{4}}$$

$f(x,y) = x \cdot \ln y$ $\quad 1 \leq y \leq 2$ $\quad 0 \leq x \leq y+1$

$$\int_{y=1}^{2} \int_{x=0}^{y+1} x \cdot \ln y \, dx \, dy \Longrightarrow \int_{1}^{2} \ln y \, dy \int_{0}^{y+1} x \, dx \quad \cdots$$

$$\Rightarrow \frac{1}{2} \int_{1}^{2} \underbrace{(y+1)^2}_{u'} \cdot \underbrace{\ln y}_{v} \, dy \quad \Rightarrow \text{partielle Integration}$$

$$\Rightarrow u = \frac{1}{3}(y+1)^3$$

$$v' = \frac{1}{y}$$

$$\Rightarrow (u \cdot v)' = u'v + uv' \Rightarrow u'v = (u \cdot v)' - uv' \Big| \int$$

$$\boxed{\int u'v = \cancel{\int (u \cdot v)'} - \int uv'}$$

$$(u \cdot v)$$

$$\int u'v = \left[\left[\frac{1}{3}(y+1)^3 \cdot \ln y\right]_1^2 - \int_1^2 \frac{1}{3}\underbrace{(y+1)^3}_{y^3+3y^2+3y+1} \cdot \frac{1}{y}\right] \cdot \frac{1}{2}$$

$\times$

$$\Rightarrow \frac{1}{2}\left[\underbrace{\left[\frac{1}{3}(y+1)^3 \cdot \ln y\right]_1^2}_{\underbrace{(2+1)^3}_{27} \cdot \ln 2} - \frac{1}{3} \cdot \int_1^2 \left(y^2+3y+3+\frac{1}{y}\right) dy\right]$$

$$- \left[\frac{1}{3}y^3 + \frac{3}{2}y^2 + 3y + \ln y\right]_1^2$$

$$- \left[\left(\frac{1}{3}\cdot 27 - \frac{1}{3}\right) + \left(\frac{3}{2}\cdot 4 - \frac{3}{2}\right) + (6-3) + (\ln(2) - 0)\right]$$

$$27 \cdot \ln 2 - \frac{26}{3} - \frac{9}{2} - 3 - \ln 2 = 26 \ln 2 - \frac{97}{6} \approx 1{,}36$$

Doppelintegral $f(x,y) = y^{1+xy} \cdot \ln y = y \cdot y^{x \cdot y} \cdot \ln y$

$$A = \left\{ x, y \,;\, \underbrace{0 \le x \le \frac{1}{y}}_{1.},\ 1 \le y \le 2 \right\}$$

Hinweis: es gilt $\int a^x \, dx = \frac{a^x}{\ln(a)} + c$

$$= \int_1^2 y \cdot \ln y \, dy \int_0^{\frac{1}{y}} y^{x \cdot y} \, dx$$

$$y^{x \cdot y} = (\overset{a}{\overset{\downarrow}{y^y}})^x \Rightarrow \int (y^y)^x dx = \frac{(y^y)^x}{\ln(y^y)}$$

$$= \frac{(y^y)^x}{y \cdot \ln(y)}$$

$$\Rightarrow \int_1^2 \cancel{y \cdot \ln y} \, dy \cdot \left[\frac{(y^y)^{\frac{1}{y}}}{\cancel{y \cdot \ln y}} - \frac{(y^y)^0}{\cancel{y \cdot \ln y}} \right]$$

$$= \int_1^2 (y-1) \, dy = \underline{\frac{1}{2}}$$

$$y' + \frac{3y}{x} = x \qquad y(1) = 2$$

Variation der Konstanten

$$y' + a(x) \cdot y = g(x) \Rightarrow \boxed{y(x) = \left(k + \int e^{A(x)} \cdot g(x)\, dx \right) \cdot e^{-A(x)}}$$

$a(x) = \frac{3}{x} \quad A(x) = 3 \cdot \ln x$

$g(x) = x$

$$\Rightarrow y(x) = \left(k + \int x^3 \cdot x\, dx \right) \cdot \frac{1}{x^3}$$

$e^{A(x)} = e^{\ln x^3} = x^3$

$e^{-A(x)} = \frac{1}{x^3}$

$$= \left(k + \frac{1}{5} x^5 \right) \frac{1}{x^3} = \frac{k}{x^3} + \frac{1}{5} x^2$$

$$\Rightarrow \frac{k}{1} + \frac{1}{5} = 2 \Rightarrow k = \frac{9}{5}$$

Sei $\underline{x = t}$ $\Rightarrow$ $s\,F(s) - f(0) + \frac{3}{t} \cdot F(s) = t \;\big|\cdot t$

$\Rightarrow$ geht nicht