Tutorium Math 2/3 23.10.25 BMC

Womit fangen wir an? **Extremwerte** Wiederholung

$$f(x,y) = x^2 + y^2 + xy - 2x + 3y + 7$$

$f_x = 2x + y - 2 = 0$ $f_y = 2y + x + 3 = 0$

$\Rightarrow y = 2 - 2x$ einsetzen $2(2-2x) + x + 3 = 0$

$4 - 4x + x + 3 = 0 \Rightarrow x = \frac{7}{3}$

damit $y = 2 - \frac{14}{3} = -\frac{8}{3} = y$

verdächtiger Punkt

$f_{xx} = 2$ $f_{yy} = 2$ $f_{xy} = 1$ $\det \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = 3 > 0 \Rightarrow$ Extr. Wert

$f_{xx} = 2 > 0 \Rightarrow$ Minimum

Doppelintegral

$$\iint_A 8xy \, dA \quad \text{für } A = \left\{ (x, y) \in \mathbb{R}^2 \,/\, 0 \le y \le 1 \,;\, y^2 \le x \le 2-y \right\}$$

$$\int_0^1 dy \int_{y^2}^{2-y} 8xy \, dx = \int_0^1 8y \, dy \left[ \tfrac{1}{2} x^2 \right]_{y^2}^{2-y} = \frac{1}{2} \int_0^1 8y \, dy \left[ (2-y)^2 - y^4 \right]$$

$$= 4 \int_0^1 y \cdot \left( 4 - 4y + y^2 - y^4 \right) dy = 4 \int_0^1 \left( 4y - 4y^2 + y^3 - y^5 \right) dy$$

$$= 4 \cdot \left[ 2y^2 - \frac{4}{3} y^3 + \frac{1}{4} y^4 - \frac{1}{6} y^6 \right]_0^1 = 4 \cdot \left[ 2 - \frac{4}{3} + \frac{1}{4} - \frac{1}{6} \right] = \underline{\underline{3}}$$

$f(x,y) = 4xy - x^2 - 1 + 2y - 5y^2$

$f_x = 4y - 2x \qquad f_y = 4x + 2 - 10y \qquad \overset{!}{=} 0$ beide

$4y = 2x \Rightarrow x = 2y$ einsetz. $4x + 2 - 10y = 0$

$\Rightarrow 8y + 2 - 10y = 0 \Rightarrow y = 1 \Rightarrow x = 2$

$f_{xx} = -2 \qquad f_{yy} = -10 \qquad f_{xy} = 4 \Rightarrow H = \begin{vmatrix} -2 & 4 \\ 4 & -10 \end{vmatrix} = 4 > 0$

$f_{xx} = -2 \Rightarrow < 0 \Rightarrow$ Maximum.

Noch ein Doppelintegral: $\iint_A 3xy\sqrt{x^2+y^2}\,dA$ mit $0 \leq x \leq 2$, $0 \leq y \leq \frac{3}{4}x$ I.

$$\Rightarrow \int_0^2 3x\,dx \int_0^{\frac{3}{4}x} y\sqrt{x^2+y^2}\,dy \qquad \Rightarrow \frac{1}{3}\sqrt{\left(y^2+x^2\right)^3}\Bigg]_0^{\frac{3}{4}x}$$

Tabelle 117

$$\Rightarrow \frac{1}{3}\sqrt{\left(\left(\frac{3}{4}x\right)^2+x^2\right)^3} - \frac{1}{3}x^3 \quad \cdots \quad \int_0^2 3x\,dx \quad \cdots = \int_0^2 \frac{61}{64}x^4 =$$

$$= \frac{1}{5}\cdot\frac{61}{64}x^5\Bigg]_0^2 = \underline{\underline{6{,}1}}$$