Math.-Tutorium BMC 2 1.4.26

Auf Wunsch der Kollegen, heute Wiederholung Ableitungen.

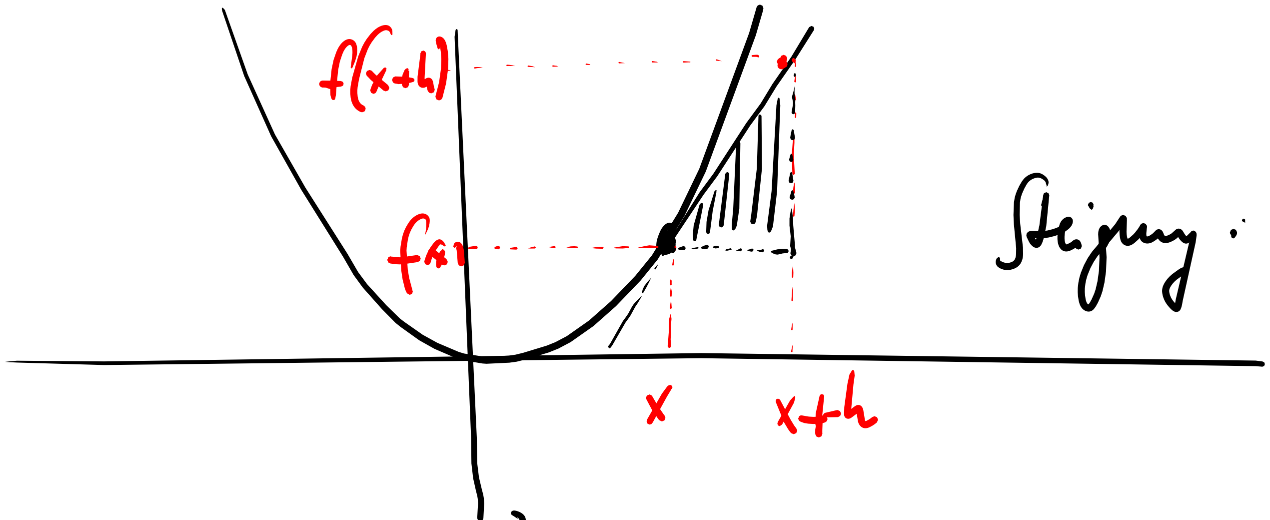
Ableitung ist eine Grenzwertangelegenheit.

Einfaches Beispiel $f(x) = y = x^2$ Herleitung d. Ableitung mit Limes

! Anscheinend gibt es Probleme mit Outlook bei Mails

$\Rightarrow$ mathcambulanz@t-online.de

$\boxed{\text{f. Wiederholer Mi ab } 10.^{00}}$



Steigung: $\dfrac{f(x+h) - f(x)}{(x+h) - x}$

Sei $f(x) = x^2$

$$\Rightarrow \frac{(x+h)^2 - x^2}{(x+h) - x} = \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{\cancel{x} + h - \cancel{x}} = \frac{2xh + h^2}{h}$$

$$= \frac{\cancel{h}(2x + h)}{\cancel{h}} = 2x + h \xrightarrow{h \to 0} = \underline{\underline{2x}}$$

Warum ist e^x abgel. wieder e^x?

$$e^x = \overset{\frac{x^0}{0!}}{1} + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \frac{x^n}{n!}$$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$$e^x = 1 + x + \frac{x^2}{2 \cdot 1} + \frac{x^3}{3 \cdot 2 \cdot 1} + \frac{x^4}{4 \cdot 3 \cdot 2 \cdot 1} + \dots$$

Anzahl Mögl. Lotto

$$e^1 = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \dots \quad 2{,}7\dots$$

$$\frac{49!}{6!\,(49-6)!}$$

$$= 0 + 1 + \frac{\cancel{2}x}{\cancel{2}} + \frac{\cancel{3}x^2}{\cancel{3} \cdot 2 \cdot 1} + \frac{\cancel{4}x^3}{\cancel{4} \cdot 3 \cdot 2 \cdot 1} + \dots$$

$43!$

Taylor bzw. McLaurin Reihe

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

<u>Übungen:</u>

1.) und 2.) Ableitung

a) $f(x) = \ln\left(2+3x^2\right)$ $\quad f'(x) = \frac{1}{(2+3x^2)} \cdot (0+6x)$

$f'(x) = \frac{6x}{2+3x^2}$ $\quad f''(x) = \frac{6\cdot(2+3x^2) - 6x\cdot 6x}{(2+3x^2)^2}$ $\quad$ Q.R. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

$u = 6x \quad u' = 6$

$v = 2+3x^2 \quad v' = 6x$

$= \frac{12+18x^2-36x^2}{(2+3x^2)^2}$

$= \frac{12-18x^2}{(2+3x^2)^2} = \frac{6(2-3x^2)}{(2+3x^2)^2}$

$f(x) = \ln\left(2x^2+x\right)$ $\quad f'(x) = \frac{1}{2x^2+x}\cdot(4x+1) = \frac{4x+1}{2x^2+x}$

$u = 4x+1 \quad u' = 4$

$v = 2x^2+x \quad v' = 4x+1$

$\Rightarrow f''(x) = \frac{4\cdot(2x^2+x) - (4x+1)\cdot(4x+1)}{(2x^2+x)^2}$

c) $f(x) = \underbrace{2x}\cdot\underbrace{\ln(4+x)}$ $\qquad f'(x) = u'v + uv'$ $\qquad u = 2x \quad u' = 2$

$v = \ln(4+x)$

$$f'(x) = 2\cdot\ln(4+x) + 2x\cdot\frac{1}{(4+x)}\cdot 1 \qquad v' = \frac{1}{(4+x)}\cdot 1$$

$$= 2\cdot\ln(4+x) + \frac{2x}{4+x}$$

$$f''(x) = \frac{2}{(4+x)} + \frac{2\cdot(4+x) - 2x\cdot 1}{(4+x)^2} = \frac{2}{4+x} + \frac{8 + \cancel{2x} - \cancel{2x}}{(4+x)^2}$$

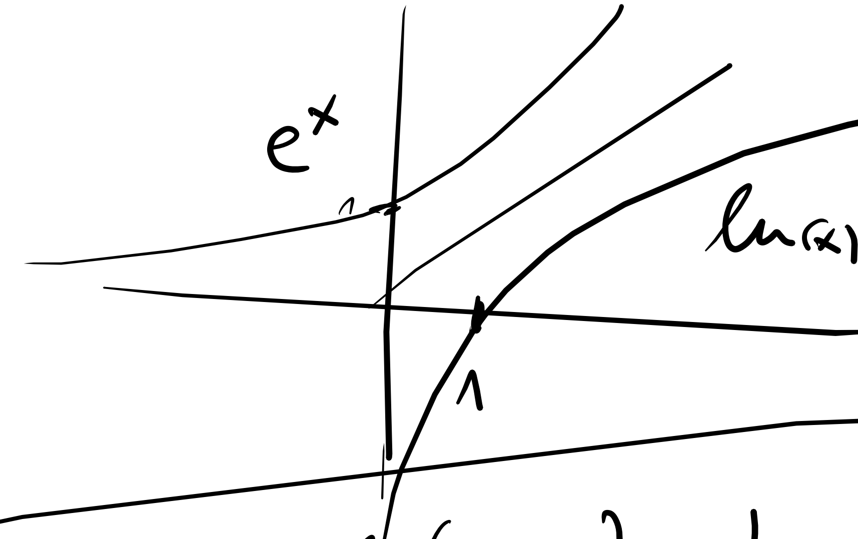
d) $f_t(x) = \ln(x^2+t) \Longrightarrow f_t'(x) = \frac{1}{x^2+t}\cdot 2x$

$$f_t'(x) = \frac{2x}{x^2+t} \qquad f_t''(x) = \frac{2(x^2+t) - 2x\cdot 2x}{(x^2+t)^2} \;---$$

evtl. vereinf.

e) $f(x) = \ln(\ln(3x+1))$

$$f'(x) = \frac{1}{\ln(3x+1)} \cdot \frac{1}{(3x+1)} \cdot 3$$



$$f'(x) = \frac{3\,(x^0)}{[\ln(3x+1)\cdot(3x+1)]^1} \qquad f''(x)$$

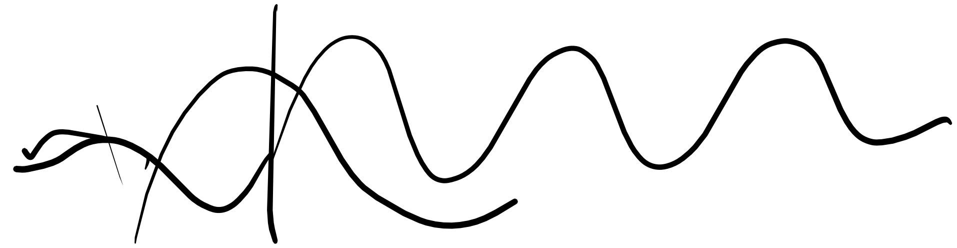
$u = \ln(3x+1) \quad u' = \frac{1}{(3x+1)}\cdot 3$

$v = 3x+1 \quad v' = 3$

$$= 3\cdot[\ln(3x+1)\cdot(3x+1)]^{-1} \Rightarrow f''(x) = -1\cdot 3\cdot[\underbrace{\ln(3x+1)}_{u}\cdot\underbrace{(3x+1)}_{v}]^{-2}$$

$$\frac{-3}{[\ln(3x+1)\cdot(3x+1)]^2}\cdot\left(\frac{3}{(3x+1)}(3x+1) + \ln(3x+1)\cdot 3\right)$$

$$f(x) = \sin(e^x + 1)$$



$$f'(x) = \underbrace{\cos(e^x + 1)}_{u} \cdot \underbrace{e^x}_{v}$$

$$u = \cos(e^x + 1) \qquad u' = -\sin(e^x + 1) \cdot e^x$$

$$v = e^x \qquad v' = e^x$$

$$f''(x) = -\sin(e^x + 1) \cdot e^x e^x + \cos(e^x + 1) \cdot e^x$$

$$= \cos(e^x + 1) \cdot e^x - \sin(e^x + 1) \cdot e^{2x}$$

$$(e^x)^2 = e^{2x} = e^{x+x}$$

$$= e^x \cdot e^x$$

$$f(x) = \sqrt[2]{2x+1} \cdot e^{-x} = \underbrace{(2x+1)^{\frac{1}{2}}}_{u} \cdot \underbrace{e^{-x}}_{v}$$

$$f'(x) = \frac{1}{2}(2x+1)^{-\frac{1}{2}} \cdot e^{-x} - (2x+1)^{\frac{1}{2}} \cdot e^{-x} \cdot \cancel{(-1)}$$

$$= \frac{e^{-x}}{2 \cdot \sqrt{2x+1}} - e^{-x} \cdot \left(\sqrt{2x+1}\right)$$