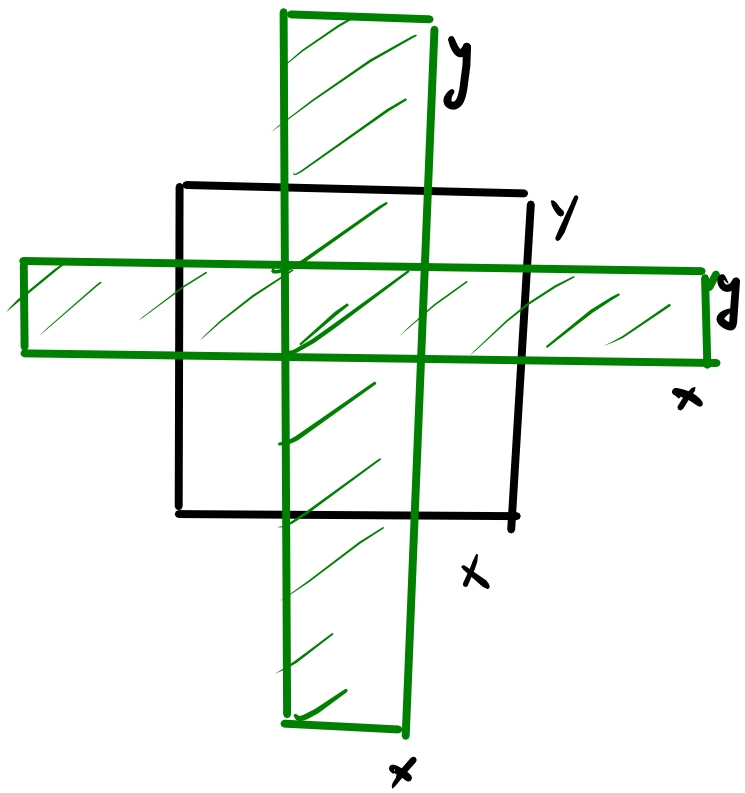
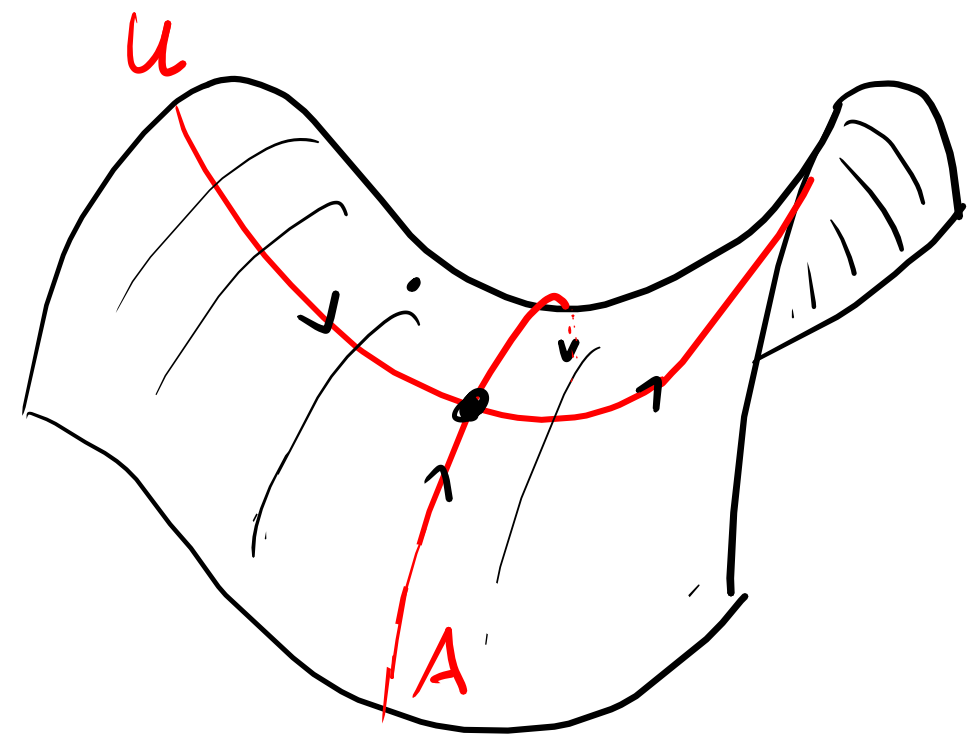
Tutorium Math. f. BMC 8.4.26





$x = 1 \quad y = 1 \qquad A = 1\,m^2 \qquad u = 4\,m$

$x = 0{,}5 \quad y = 2 \qquad A = 1\,m^2 \qquad u = 5\,m$

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anders gerechnet

$u = 2x + 2y$ $\quad x = \frac{u-2y}{2} \Rightarrow A = \frac{u-2y}{2} \cdot y = \frac{u \cdot y - 2y^2}{2}$

$A = x \cdot y$

$A = \frac{ux - 2x^2}{2}$ $\quad \frac{dA}{dx} = \frac{u-4x}{2} = 0 \Rightarrow u = 4x$

$\Rightarrow \frac{d^2A}{dx^2} = \frac{-4}{2} = -2 < 0 \Rightarrow$ Max

$4x = 2x + 2y$

$2x = 2y \Rightarrow x = y$

Quadrat

Bestimmen Sie alle lok. Extrema und Sattelpkt. der Fkt:

$$f(x,y) = e^x(x^2+y^2)$$

$$\frac{\partial f}{\partial x} = f_x = e^x x^2 + 2x\cdot e^x + e^x \cdot y^2 = 0 \quad \text{(notwendig)}$$

$$\frac{\partial f}{\partial y} = f_y = 2ye^x = 0$$

$$\Rightarrow 2y = 0 \Rightarrow y = 0$$

$$x^2 + 2x = 0$$

$$\Rightarrow x(x+2) = 0$$

$$\Rightarrow (x_1, y_1) = (0,0) \qquad (x_2, y_2) = (-2, 0)$$

Krit. Punkte

$$\frac{\partial^2 f}{\partial x^2} = e^x \cdot x^2 + 4x \cdot e^x + 2e^x + e^x y^2$$

$$\frac{\partial^2 f}{\partial y^2} = 2e^x \qquad \frac{\partial^2 f}{\partial x \partial y} = 2y \cdot e^x$$

D für $(x_1, y_1) \Rightarrow D = 2 \cdot 2 = 4 > 0 \Rightarrow$ Extrema

$$\frac{\partial^2 f}{\partial x^2} = 2 > 0 \Rightarrow \underline{\text{Min.}}$$

D für $(x_2, y_2) \Rightarrow D = -2 \cdot e^{-2} \cdot 2 \cdot e^{-2} = -4 \cdot e^{-4} < 0$

$\Rightarrow$ Sattelpkt.

Gegeben $f(x,y) = e^{x+1}(xy - y + 2)$ Extr. / Sattelpkt.

↳ $x \cdot y \cdot e^{x+1} - y e^{x+1} + 2e^{x+1}$

$f_x = y \cdot e^{x+1} \cdot x + y \cdot e^{x+1} - y \cdot e^{x+1} + 2e^{x+1} = 0$

$f_y = x \cdot e^{x+1} - e^{x+1} = 0 \Rightarrow e^{x+1}(x-1) \Rightarrow \underline{\underline{x = 1}}$ Krit.

mit $f_x \Rightarrow e^{x+1}(xy + y - y + 2) = 0 \Rightarrow x \cdot y = -2 \Rightarrow \underline{\underline{y = -2}}$

$f_{xx} = y \cdot e^{x+1} + xy \cdot e^{x+1} + y \cdot e^{x+1} - y \cdot e^{x+1} + 2e^{x+1} \Rightarrow -2e^2$

$f_{yy} = 0$

$f_{xy} = e^2 \Rightarrow \begin{vmatrix} -2e^2 & e^2 \\ e^2 & 0 \end{vmatrix} = -e^4 < 0 \Rightarrow$ Sattel

Korrektur:

$$f_{xx} = y \cdot e^{x+1} + xy \cdot e^{x+1} + y e^{x+1} - y e^{x+1} + 2e^{x+1}$$

$$= -2 \cdot e^2 - 2 \cdot e^2 + 2 \cdot e^2 = -2e^2$$

Bestimmen Sie von $f(x,y) = 10 - 2y + x^2 + y^2 - \frac{x^4}{2}$ Extrem-/Sattel

$f_x = 2x - 2x^3 = 0 \Rightarrow x(2 - 2x^2) = 0 \Rightarrow x = 0$ und $x = \pm 1$

$f_y = -2 + 2y = 0 \Rightarrow y = 1$

$f_{xx} = 2 - 6x^2 \quad f_{yy} = 2 \quad f_{xy} = f_{yx} = 0$

$$\det H_{(1)} = \begin{vmatrix} 2-6x^2 & 0 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 2-6 & 0 \\ 0 & 2 \end{vmatrix} = -8 < 0 \quad \text{Sattel}$$

$$\det H(-1) = \text{"} \quad \text{"} \quad \text{"} \quad = -8 < 0 \quad \text{"}$$

$$\det H(0) = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0 \qquad f_{xx}(0) = 2 > 0 \Rightarrow \underline{\text{Min.}}$$

Bestimmen Sie für $f(x,y) = (x+1)\cdot(x^2+5x-5+y^2)$ Ext./Sattel.

$$\Rightarrow x^3 + 5x^2 - 5x + xy^2 + x^2 + 5x - 5 + y^2$$

$$f(x,y) \to x^3 + 6x^2 + xy^2 + y^2 - 5$$

$f_x = 3x^2 + 12x + y^2 = 0 \Rightarrow f_{xx}$

$f_y = 2xy + 2y = 0 \Rightarrow 2xy = -2y \Rightarrow x = -1$ f_{yy}, f_{xy}.

in $f_x = 3 - 12 + y^2 = 0 \Rightarrow y^2 = 9 \Rightarrow y = \pm 3$

Ziel: Min $(0,0)$ Max $(-4,0)$

Sattel $(-1,3)$ Sattel $(-1,-3)$