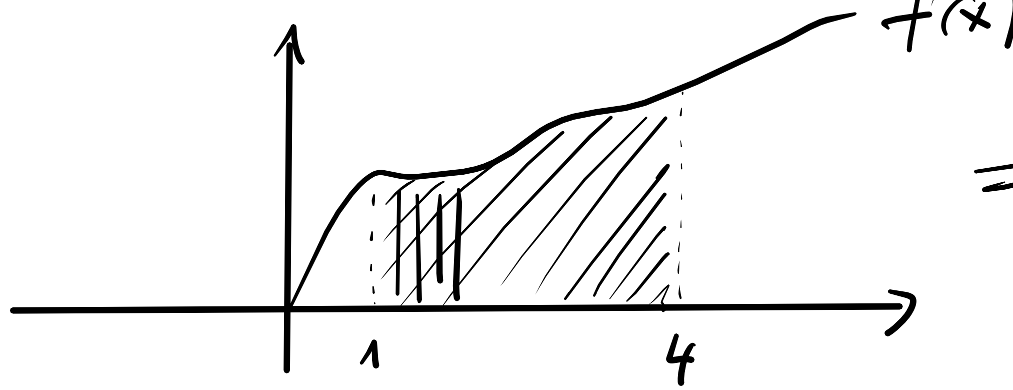
Math 1/2 f. BMC 8.4.26

Wo liegen die Probleme – bitte Wünsche

Integralrechnung:



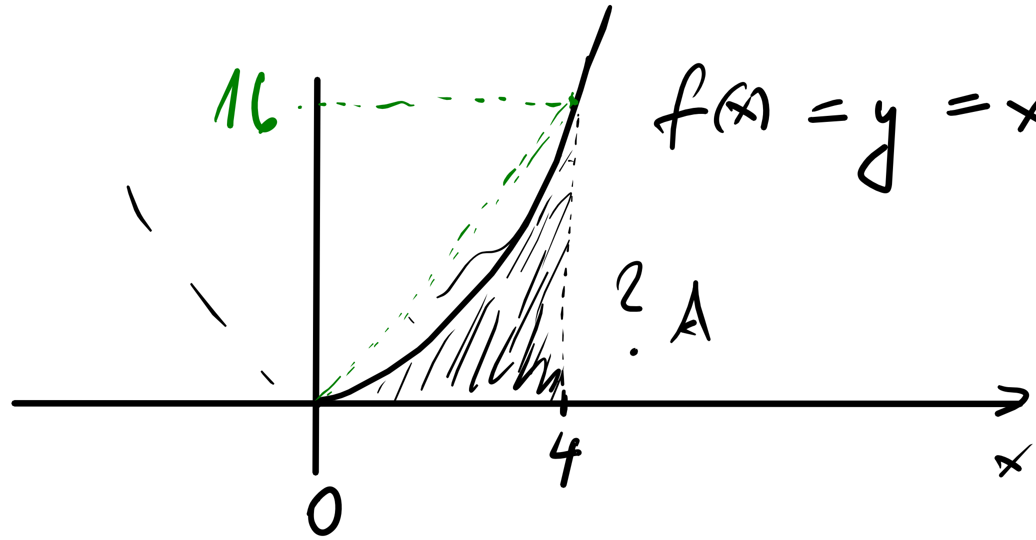
$\Rightarrow \int f(x)\,dx \ \hat{=}$ Fläche unter dem Graphen.

Einfachste Regel:

"n" kann alles sein.

$\int f(x) = \int x^3\,dx = \frac{1}{4}x^4 = \int x^n\,dx = \underline{\underline{\frac{1}{n+1}\cdot x^{n+1}}}$

Bsp. $\Rightarrow n = 5 \Rightarrow \int x^5\,dx = \frac{1}{5+1}x^{5+1} = \underline{\underline{\frac{1}{6}\cdot x^6}}$



$f(x) = y = x^2$

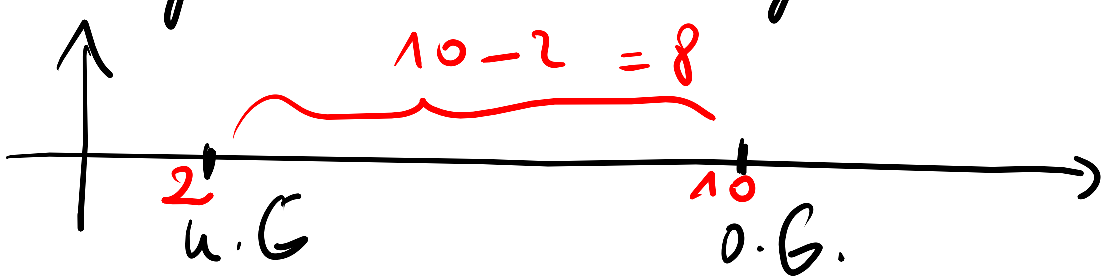
Fläche ungef. $\frac{1}{2}\, 4 \cdot 16 \approx \underline{\underline{32}}$

obere Grenze

$\Rightarrow \int_0^4 x^2 \, dx$ mit Formel $\int x^n \, dx \Rightarrow \left[\frac{1}{3} x^3 \right]_0^4 \Rightarrow \frac{1}{3} 4^3 - \frac{1}{3} \cdot 0^3 = \frac{64}{3} \approx \underline{\underline{21{,}3}}$

untere Grenze

$\Rightarrow$ ob. Grenze − untere Grenze



Finden Sie die Stammfunktionen von $\int$

$$f(x) = \cancel{2}x^3 + \frac{5}{2\cdot x^{a+1}} - 7\sqrt[6]{x^5} + a^3 + 2\cdot\sin(1-2x)$$

$$\int f(x) = 2\cdot(x^3) = 2\cdot\left(\frac{1}{4}x^4\right) = \frac{2}{4}x^4 = \underline{\underline{\frac{1}{2}x^4 + c}} \qquad \frac{1}{2}x^4 \xrightarrow{\text{abl.}} = \frac{4}{2}\cdot x^3 = \underline{\underline{2x^3}}$$

$$\frac{1}{x^2} = x^{-2}$$

$$= \frac{5}{2}\cdot\frac{1}{x^{a+1}} = \frac{5}{2}\cdot\left(x^{a+1}\right)^{-1} = \frac{5}{2}x^{-a-1} \xrightarrow{\int} \frac{5}{2(-a-(1+1))}\cdot x^{-a-\cancel{1}\,\cancel{+1}}$$

$$(x^a)^3 = x^{3a}$$

$$= \frac{5}{2}\cdot\frac{1}{-a}\cdot x^{-a} = \frac{5\cdot x^{-a}}{2\cdot(-a)} = \frac{5}{2}\cdot\frac{1}{-a\cdot x^a} = -\frac{5}{2a\cdot x^a}$$

n war $-(a+1)$

$$\int -7 \cdot \sqrt[6]{x^5}\, dx$$

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$-7 \int \left(x^5\right)^{\frac{1}{6}} \cdot dx = -7 \cdot \int x^{\frac{5}{6}}\, dx$$

$$\sqrt[3]{a^1} = a^{\frac{1}{3}} \quad \Big|\text{ hoch } 3$$

$$a = \left(a^{\frac{1}{3}}\right)^3 = a^1 = \underline{\underline{a}}$$

$$= -7 \cdot \frac{1}{\underbrace{\frac{5}{6}+1}_{\frac{5}{6}+\frac{6}{6}}}\, x^{\frac{5}{6}+1} = -7 \frac{1}{\frac{11}{6}} \cdot x^{\frac{11}{6}}$$

$$= -7 \cdot \frac{6}{11} \cdot x^{\frac{11}{6}} = \frac{-42}{11} \cdot \left(x^{11}\right)^{\frac{1}{6}}$$

$$= -\frac{42}{11} \cdot \sqrt[6]{x^{11}}$$

$$\int a^3\, dx = \int a^3 \cdot 1 = \int a^3 \cdot x^0\, dx = a^3 \int x^0\, dx$$

$$= a^3 \frac{1}{0+1} x^{0+1} = a^3 \cdot 1 \cdot x^1 = \underline{\underline{a^3 \cdot x}}$$

$$\int \underbrace{2 \cdot \sin(1-2x)} \, dx$$

$$2 \int \sin(1-2x)\, dx = \underset{-2}{2} \cdot \left[-\cos(1-2x) \right]$$

fertig? nein

muss weg

$\sin(x) \xrightarrow{\text{abl.}} \cos(x)$, $\xleftarrow{\text{Stammf.}}$

$\cos(x) \xrightarrow{\text{abl.}} -\sin(x)$, $\xleftarrow{\text{Stammf.}}$

$-\sin(x) \xrightarrow{\text{abl.}} -\cos(x)$, $\xleftarrow{\text{Stammf.}}$

$-\cos(x) \xrightarrow{\text{abl.}} \sin(x)$, $\xleftarrow{\text{Stammf.}}$

Probiere Ableitg.

$$\Longrightarrow \quad 2 \cdot \sin(1-2x) \cdot (-2)$$

$$\int \Longrightarrow \boxed{\cos(1-2x)} \checkmark \quad -2$$

Probiere jetzt: ableiten $\Longrightarrow -\sin(1-2x) \cdot (-2) = \sin(1-2x)$ ✓

partielle Integration

Produktregel

$$(u \cdot v)' = u'v + uv' \quad \Big| \int$$

$$\underbrace{\int (u \cdot v)'} = \int u'v + \int uv'$$

$$u \cdot v = \int u'v + \int uv' \Rightarrow$$

$$\boxed{\Rightarrow \int uv' = u \cdot v - \int u'v}$$

Beispiel

$$\int \underset{u}{x} \cdot \underset{v'}{\sin(x)}\, dx$$

$u = x \qquad v' = \sin(x)$

$u' = 1 \qquad v = -\cos(x)$

benutze Formel $\Rightarrow$

$$\Rightarrow \int x \cdot \sin(x)\, dx = \underset{u}{x} \cdot \underset{v}{(-\cos(x))} - \int 1 \cdot (-\cos(x))\, dx$$

$$= \underline{-x \cdot \cos(x)} + \int \cos(x)\, dx = \underline{\underline{-x \cdot \cos(x) + \sin(x)}}$$

$$\int_0^1 \underset{u}{(1+2x)} \cdot \underset{v'}{e^{-x}}\,dx$$

wenn $u = 1+2x \quad u' = 2$

$v = -1 \cdot e^{-x} \quad v' = e^{-x}$

$= -e^{-x}$

$$\int uv' = u \cdot v - \int u'v \quad = \int_0^1 (1+2x) \cdot e^{-x}\,dx = \underline{(1+2x)(-e^{-x})} - \int 2 \cdot (-e^{-x})\,dx$$

$$= -e^{-x}(1+2x)\Big]_0^1 + 2\int_0^1 e^{-x}\,dx$$

$$= -e^{-x}(1+2x)\Big|_0^1 - 2e^{-x}\Big|_0^1 = \left[-e^{-1}(1+2\cdot 1)\right] - \left[-e^{0}(1+2\cdot 0)\right] = -e^{-1}(1+2) - 1$$

$$\left[-2 \cdot e^{-1} - -2e^{0}\right] = -\frac{2}{e} + 1$$