Tutorium BMC2 Math. 15.4.26

Aufgabe fertig machen: war $f(x,y) = (x+1)(x^2+5x-5+y^2)$

$$f_x = 3x^2 + 12x + y^2 = 0 \xrightarrow{x=-1} 3 - 12 + y^2 = 0 \Rightarrow \underline{\underline{y = \pm 3}}$$

$$f_y = 2xy + 2y = 0 \Rightarrow 2x\cancel{y} = -2\cancel{y} \rightarrow \underline{\underline{x = -1}}$$

$$f_x = 3x(x+4) + y^2 = 0$$

mit $f_y = 2y(x+1) = 0 \Rightarrow \underline{\underline{y = 0}}$ in $f_x = 3x(x+4) + 0 = 0$

$$\Rightarrow \underline{\underline{x = 0}} \wedge \underline{\underline{x = -4}}$$

$$f_{xx} = 6x + 12$$

$$f_{yy} = 2x + 2, \quad f_{xy} = 2y$$

$$H(-1,3) = \begin{vmatrix} 6x+12 & 2y \\ 2y & 2x+2 \end{vmatrix} = \begin{vmatrix} 6 & 6 \\ 6 & 0 \end{vmatrix} = -36 < 0 \Rightarrow$$

Sattelpkt.

$$H(-1,-3) = \begin{vmatrix} 6 & -6 \\ -6 & 0 \end{vmatrix} = -36 \Rightarrow < 0 \Rightarrow \text{Sattelpkt.}$$

$$H(0,0) = \begin{vmatrix} 12 & 0 \\ 0 & 2 \end{vmatrix} = 24 > 0 \Rightarrow \text{Extr.!}$$

$$f_{xx}(0,0) = 12 > 0 \Rightarrow \underline{\underline{\text{Min!}}}$$

$$H(-4,0) \quad \ldots\ldots \quad D > 0 \Rightarrow \underline{\underline{\text{Max.}}}$$

$$f_{xx} < 0$$

Berechnen Sie die Ableitung von $f(x) = x^{x^x} \Longrightarrow$

$$x^{x^x} = e^{\ln(x^{x^x})} = e^{x^x \cdot \ln(x)} \overset{\text{abl.}}{\Longrightarrow} \underbrace{e^{\ln x^{x^x}}}_{x^{x^x}} \cdot \left(x^x \cdot (\ln(x)+1)\cdot \ln(x) + \underbrace{x^x \cdot \frac{1}{x}}_{x^{x-1}}\right)$$

Zw.R.

$$(x^x)' = e^{\ln x^x} = e^{x \cdot \ln x}$$

$$\overset{\text{abl.}}{\Longrightarrow} \underbrace{e^{x\cdot\ln(x)}}_{x^x} \cdot (\ln(x) + 1)$$

Zeigen Sie daß der Punkt $(\pi, -1)$ ein lok. Extremwert der Funktion:

$$f(x,y) = \cos(x^2)\cdot\left(\frac{y^3}{3} - y\right)$$ ist.

ausmult. $\Rightarrow \frac{y^3}{3} \cdot \cos(x^2) - y \cdot \cos(x^2)$

$\frac{\partial}{\partial x} \Rightarrow -\frac{y^3}{3} \sin(x^2) \cdot 2x \; \cancel{-} + y \cdot \sin(x^2) \cdot 2x \overset{!}{=} 0$ prüfe mit $x = \sqrt{\pi}$

$(\sqrt{\pi})^2 = \pi \qquad (\sqrt{\pi})^2 = \pi$

$\sin \pi = 0$!

$\frac{\partial}{\partial y} \Rightarrow y^2 \cdot \cos(x^2) - \cos(x^2)$

$\pi \qquad \pi$

$\underbrace{(-1)^2}_{1} \quad (-1) \quad - (-1)$

$-1 \qquad 1 \qquad = 0$ ✓

kann Extremum sein.

Prüfen ob Minimum ?!

$$f_{yy} = 2y \cdot \underbrace{\cos(x^2)}_{-1} = -2y = \underline{\underline{2}}$$

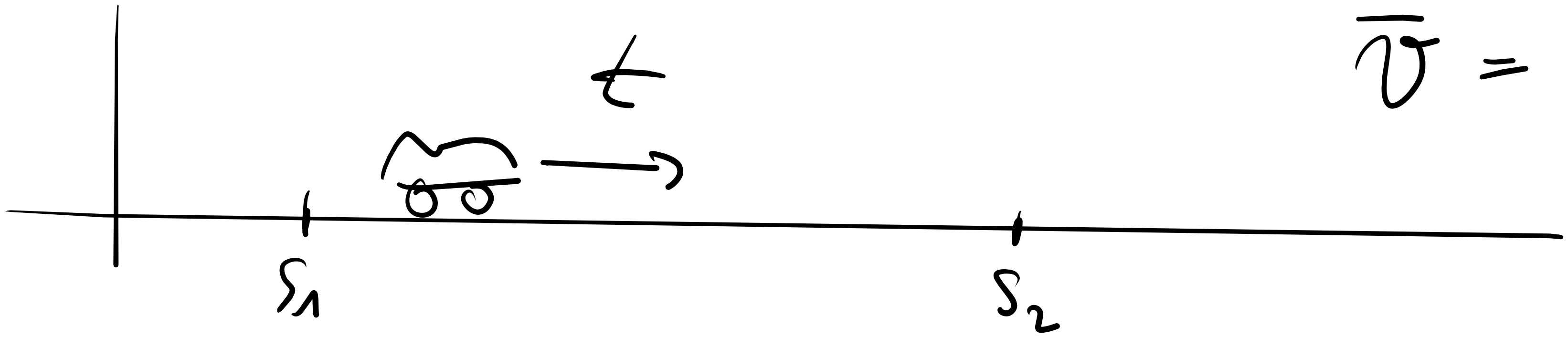
$$f_{yx} = y^2 \cdot -\sin(x^2) \cdot 2x = -\sin(x^2) \cdot 2x$$

f_{xx} . — — — — $D > 0$ ist

$f_{xx}(P) > 0 \Longrightarrow$ Minimum.

Diff. Gleichungen 1. Ordnung: Separationsansatz $\hat{=}$ Trennung der Variablen

$$f'(x,y) = f(x,y) \Longrightarrow f(x)\,dx = f(y)\,dy$$



$$\bar{v} = \frac{s}{t}$$

$$v_\Delta = \frac{\Delta s}{\Delta t}$$

$$\Rightarrow v_{mom} = \frac{ds}{dt} = \dot{s}$$

Beispiel:

$$y' = \sqrt{\frac{y+1}{x}} \qquad y' \overset{!}{=} \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{y+1}{x}} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{y+1}}{\sqrt{x}} \Rightarrow \frac{1}{\sqrt{y+1}} \cdot dy = \frac{1}{\sqrt{x}}\, dx \;\Big|\int$$

$$= \int \frac{1}{\sqrt{y+1}}\, dy = \int \frac{1}{\sqrt{x}}\, dx$$

$$(y+1)^{-\frac{1}{2}}\, dy = x^{-\frac{1}{2}}\, dx$$

$$\boxed{\int x^n = \frac{1}{n+1} \cdot x^{n+1}}$$

$$2(y+1)^{\frac{1}{2}} + c_1 = 2 \cdot x^{\frac{1}{2}} + c_2$$

$$c_1 - c_2 = C$$

$$2C = k \;---$$

$$\cancel{2}(y+1)^{\frac{1}{2}} = \cancel{2} \cdot x^{\frac{1}{2}} + k$$

$$\sqrt{y+1} = (\sqrt{x} + k) \quad | \text{ quadr.}$$

$$y+1 = x + 2\sqrt{x} \cdot k + k^2 \implies y(x) = x + 2\sqrt{x} \cdot k + k^2 - 1$$

Anfangswertproblem $y(0) = 2$ $\quad \boxed{y' = (1-y) \cdot x^2}$

$$y' = \frac{dy}{dx} = (1-y)\cdot x^2 \implies \int \frac{1}{1-y}\,dy = \int x^2\,dx$$

$$y(0) = 2$$

f. Probe $\implies -\ln(1-y) = \frac{1}{3}x^3 + C$

f. Probe $\implies \ln(1-y) = -\frac{1}{3}x^3 - C \quad | e^{\sim}$

$$1-y = e^{-\frac{1}{3}x^3 - C}$$

$$e^{-\frac{1}{3}x^3} e^{-C}$$

$$\implies 1-y = k\cdot e^{-\frac{1}{3}x^3}$$

$$k \quad y = 1 - k\cdot e^{-\frac{1}{3}x^3}$$

$$\implies 2 = 1 - k\cdot e^0 \implies$$

$$\implies y' =$$