$$\int_0^{\sqrt{\frac{\pi}{2}}} x\cdot\Big(\overbrace{\sin(x^2)}^{u}\Big)^2\cdot\cos(x^2)\,dx$$

$$\boxed{u=\sin(x^2)} \implies \frac{du}{dx}=\cos(x^2).2x$$

$$\implies dx=\frac{du}{\cos(x^2)\cdot 2x}$$

$$\implies \int_0^{\sqrt{\frac{\pi}{2}}} x\cdot u^2\cdot\cos(x^2)\cdot\frac{du}{\cos(x^2)\cdot 2x}$$

Grenzen:

$$\underline{\underline{x_1}}=0 \Longrightarrow \underline{\underline{u_1}}=\sin(0)=0$$

$$x_2=\sqrt{\frac{\pi}{2}} \implies \underline{\underline{u_2}}=\sin\left(\left(\frac{\pi}{2}\right)\right)\underline{\underline{=1}}$$

$$\int_0^1 \frac{\cancel{x}\cdot u^2\cdot\cancel{\cos}(x^2)\cdot du}{\cancel{\cos}(x^2)\cdot 2\cancel{x}}=\frac{1}{2}\cdot\int_0^1 u^2\,du$$

$$=\frac{1}{2}\cdot\frac{1}{3}u^3\Big|_0^1=\frac{1}{2}\cdot\frac{1}{3}\cdot-0=\underline{\underline{\frac{1}{6}}}$$

andere Substitution

$$\int_0^{\sqrt{\frac{\pi}{2}}} x \cdot \left(\sin(x^2)\right)^2 \cdot \cos(x^2)\, dx$$

$$u = \left(\sin(x^2)\right)^2 \Rightarrow \sqrt{u} = \sin(x^2)$$

$$\frac{du}{dx} = 2 \cdot \sin(x^2) \cdot \cos(x^2) \cdot 2x$$

$$dx = \frac{du}{4 \cdot \sin(x^2) \cdot \cos(x^2) \cdot x}$$

$$\Rightarrow \int_0^1 \frac{\cancel{x} \cdot u \cdot \cancel{\cos(x^2)} \cdot du}{4 \cdot \sin(x^2) \cdot \cancel{\cos(x^2)} \cdot \cancel{x}} = \int_0^1 \frac{\overset{\sqrt{u}\cdot\sqrt{u}}{u} \cdot du}{4 \cdot \underbrace{\sin(x^2)}_{\sqrt{u}}}$$

$$u_1 = 0 \qquad u_2 = 1$$

$$= \frac{1}{4} \int_0^1 \sqrt{u}\, du \xrightarrow{\sqrt{u} = u^{\frac{1}{2}}} \frac{1}{4} \cdot \frac{2}{3} \cdot u^{3/2} \Big|_0^1 = \frac{2}{3} \cdot \frac{1}{4} = \frac{1}{6}$$

Berechnen Sie durch Substitution: $\int_1^2 4x \cdot e^{x^2}\,dx$

$$\boxed{u = x^2} \Rightarrow \frac{du}{dx} = 2x \qquad dx = \frac{du}{2x}$$

$$x_1 = 1 \Rightarrow u_1 = 1$$
$$x_2 = 2 \Rightarrow u_2 = 4$$

$$\Rightarrow \int_1^4 4\cancel{x} \cdot e^u \frac{du}{2\cancel{x}} = 2 \cdot \int_1^4 e^u\,du$$

$$\Rightarrow 2 \cdot e^u \Big|_1^4 = 2\left(e^4 - e^1\right) \approx 103{,}76$$

$$\int \sin(x) \cdot e^{2\cdot\cos(x)+1}\,dx \qquad \boxed{u = 2\cdot\cos(x)+1} \qquad \frac{du}{dx} = -2\cdot\sin(x)$$

$$\Rightarrow \int \cancel{\sin}(x)\, e^u \frac{du}{-\cancel{\sin}(x)\cdot 2} \Rightarrow -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + c \xRightarrow{\text{rücksubst.}}$$

$$\Rightarrow -\frac{1}{2}e^{2\cos(x)+1} + c$$

$$\int \sin(x)\cdot e^{2\cos(x)+1}\,dx \qquad u=\sin(x) \qquad \frac{du}{dx}=\cos(x)$$

$$\Rightarrow \int u\cdot e^{2\cos(x)+1}\cdot\frac{du}{\cos(x)} \quad ? \quad \Longrightarrow \text{falsche Wahl}$$

$$\int \sin(x)\, e^{2\cos(x)+1}\,dx \qquad \text{sei } \boxed{u=2\cdot\cos(x)} \qquad \frac{du}{dx}=-2\sin(x)$$

$$\Rightarrow \int \cancel{\sin(x)}\cdot e^{u+1}\cdot\frac{du}{-2\cdot\cancel{\sin(x)}} \Rightarrow -\frac{1}{2}\int e^{u+1}\,du$$

$$\Rightarrow -\frac{1}{2}\cdot e^{u+1}+c \xRightarrow{\text{rück.}} -\frac{1}{2}\cdot e^{2\cos(x)+1}+c$$

$$f(x) = \int \frac{2x^2-6}{\sqrt{x^3-9x}}\,dx \qquad u = x^3 - 9x \qquad \frac{du}{dx} = 3x^2 - 9$$

$$= \int \frac{2x^2-6}{\sqrt{u}} \cdot \frac{du}{3x^2-9} \Rightarrow \int \frac{2\cdot\cancel{(x^2-3)}}{\sqrt{u}\;3\cdot\cancel{(x^2-3)}}\,du = \frac{2}{3}\int \frac{1}{\sqrt{u}}\,du$$

$$= \frac{2}{3}\int u^{-\frac{1}{2}}\,du = \frac{2}{3}\cdot 2\cdot u^{\frac{1}{2}} = \frac{4}{3}\sqrt{u} + c \xrightarrow{\text{rücksub.}} \underline{\underline{\frac{4}{3}\sqrt{x^3-9x} + c}}$$

Gauß-Formel $\sum_{0}^{n} n = \frac{n\cdot(n+1)}{2} \Rightarrow \sum_{0}^{50} n = \frac{50(51)}{2} = \underline{\underline{1275}}$

$n \to \infty \qquad n \approx n+1 \Rightarrow \frac{n\cdot n}{2} = \underline{\underline{\frac{1}{2}n^2}}$

$$\int \frac{\sqrt{\ln(x)}}{x}\,dx \qquad u = \ln(x) \qquad \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x \cdot du$$

$$\Rightarrow \frac{\sqrt{u}}{\cancel{x}} \cdot \cancel{x} \cdot du = \int \sqrt{u}\,du = \int u^{\frac{1}{2}}\,du$$

$$= \frac{2}{3} u^{\frac{3}{2}} + c \Rightarrow \frac{2}{3}\left(\ln(x)\right)^{\frac{3}{2}} + c$$

alternativ:

$$u = \sqrt{\ln(x)} \qquad \frac{du}{dx} = \frac{1}{2}\left(\ln(x)\right)^{-\frac{1}{2}} \cdot \frac{1}{x} \Rightarrow dx$$

$$\Rightarrow \int \frac{u \cdot 2 \cdot \overbrace{\sqrt{\ln(x)}}^{u} \cdot \cancel{x}\,du}{\cancel{x}} = 2\int u^2\,du = 2 \cdot \frac{1}{3} u^3 + c$$

$$= \frac{2}{3}\left(\sqrt{\ln(x)}\right)^3 + c = \frac{2}{3}\left(\ln(x)\right)^{\frac{3}{2}} + c$$