Tutorium M2/M3 f. BUC 13.11.25

Aufgabe vom letztenmal fertig machen:

$$y_p = y_{p1} + y_{p2} = Ax \cdot e^x + Bx + C$$

$$y_p' = A \cdot e^x + e^x \cdot A \cdot x + B = A \cdot e^x + Axe^x + B$$

$$y_p'' = A \cdot e^x + A \cdot e^x + e^x \cdot Ax = 2Ae^x + Ax \cdot e^x$$

$\Rightarrow$ in die inh. DGL $\Rightarrow y'' + 2y' - 3y = 4e^x + 6x - 10$

$$\Rightarrow 2Ae^x + Axe^x + 2(A \cdot e^x + Axe^x + B) - 3(Ax \cdot e^x + Bx + C) = 4e^x + 6x - 10 \Rightarrow$$

$$2A \cdot e^x + Ax \cdot e^x + 2Ae^x + 2Ax \cdot e^x + 2B - 3Ax \cdot e^x - 3Bx - 3C = 4 \cdot e^x + 6x - 10 \Rightarrow$$

$$\Rightarrow 4A \cdot e^x - 3Bx + (2B - 3C) = 4 \cdot e^x + 6x - 10$$ mit $4A = 4 \Rightarrow$

$$\underline{\underline{A = 1}}$$

$\Rightarrow$

$-3B = 6 \Rightarrow \underline{\underline{B = -2}}$; $2B - 3C = -10 \Rightarrow \underline{\underline{C = 2}}$

$$\Rightarrow \underline{\underline{y_p = x \cdot e^{x} - 2x + 2}}$$

$\Rightarrow$ allg. Lösung der DGL: $y = y_h + y_p = \underline{\underline{C_1 \cdot e^{x} + C_2 \cdot e^{-3x} + x \cdot e^{x} - 2x + 2}}$;

Bestimmen Sie die Lösung des AWP $\quad \dot{x} + 6x \cdot \dfrac{e^{2t}}{1 + 3 \cdot e^{2t}} = e^{-3t}$; $x(0) = -1$

$\Rightarrow$ Variation der Konstanten bietet sich an!

$$a(t) = \frac{6 \cdot e^{2t}}{1 + 3 e^{2t}} \; ; \; g(t) = e^{-3t} \; ; \; A(t) = \ln\left(1 + 3 \cdot e^{2t}\right) \; ;$$

$$e^{A(t)} = 1 + 3 \cdot e^{2t} \; ; \; e^{-A(t)} = \frac{1}{1 + 3 \cdot e^{2t}}$$

$$\Longrightarrow \quad x(t) = \left(k + \int \left(1 + 3\cdot e^{2t}\right)\cdot e^{-3t}\,dt\right)\cdot \frac{1}{1+3\cdot e^{2t}}$$

$$= \left(k + \int \left(e^{-3t} + 3e^{2t}\cdot e^{-3t}\right)dt\right)\cdot \frac{1}{1+3\cdot e^{2t}}$$

$$= \left(k + \int \left(e^{-3t} + 3e^{-t}\right)dt\right) \frac{1}{1+3e^{2t}} \Rightarrow x(t) = \left(k - \frac{1}{3}e^{-3t} - 3e^{-t}\right)\cdot \frac{1}{1+3e^{2t}}$$

$$x(t) = \frac{k - \frac{1}{3}e^{-3t} - 3e^{-t}}{1+3e^{2t}} \quad \text{mit } x(0) = -1 \Longrightarrow -1 = \frac{k - \frac{1}{3} - 3}{1+3}$$

$$-4 = k - \frac{10}{3} \Longrightarrow k = -\frac{2}{3}$$

Lösen Sie das AWP:

$$y^2\left(x^2+1\right)\cdot y' = \left(1-y^3\right)\cdot x \quad \text{mit } y(0) = 2 \quad \Longrightarrow$$

$$\Rightarrow \frac{y^2}{1-y^3}\cdot\frac{dy}{dx} = \frac{x}{1+x^2} \quad \Rightarrow \frac{y^2}{1-y^3}\,dy = \frac{x}{1+x^2}\,dx \text{, zu integrieren} \Rightarrow$$

$$\Rightarrow -\frac{1}{3}\ln|y^3-1| = \frac{1}{2}\ln(1+x^2) + c$$

$$\Rightarrow \ln|y^3-1| = -\frac{3}{2}\ln(1+x^2) = \ln(1+x^2)^{-\frac{3}{2}} + c \quad \Big/ \, e^{\sim}$$

$$y^3-1 = (1+x^2)^{-\frac{3}{2}}\cdot\underbrace{e^c}_{\hat{=}\, C} \quad \Rightarrow y^3 = \frac{C}{(1+x^2)^{\frac{3}{2}}} + 1$$

$$\Rightarrow y = \sqrt[3]{\frac{C}{(1+x^2)^{\frac{3}{2}}} + 1} \quad \text{mit } y(0) = 2 \Rightarrow 2 = \sqrt[3]{\frac{C}{1} + 1}$$

$$\Rightarrow 8 = C + 1 \Rightarrow \underline{\underline{C = 7}}$$

$$y' = \frac{x y^2}{1+x^2} \qquad y(0) = 1 \implies \frac{1}{y^2}\,dy = \frac{x}{1+x^2}\,dx$$

$$\implies -\frac{1}{y} = \frac{1}{2}\ln(1+x^2) + c \implies y = \frac{-1}{\frac{1}{2}\ln(1+x^2)+c} \implies 1 = \frac{-1}{c}$$

$$\implies \underline{\underline{c = -1}}$$

$$\implies y = \frac{-1}{\frac{1}{2}\ln(1+x^2) - 1}$$ oder $$y = \frac{1}{1 - \ln\sqrt{1+x^2}}$$