Tutorium BMC 2 Mathe 29.4.26

DGL $\rightarrow$ Trennung der Variablen oder Separationsansatz.

Berechnen Sie das AWP mit $x \geq 1$

$$y' = \frac{2 \cdot e^{-2y} x^4 + e^{-2y} \cdot \sqrt{x}}{x} \quad , \quad y(1) = 0$$

$x^{-\frac{1}{2}}$

$$\Rightarrow y' = e^{-2y}\left(2x^3 + \frac{1}{\sqrt{x}}\right) \Rightarrow e^{2y}\, dy = \left(2x^3 + \frac{1}{\sqrt{x}}\right) dx \quad \Big| \int$$

$\frac{dy}{dx}$

$$\frac{1}{2} e^{2y} = \frac{1}{2} x^4 + 2\sqrt{x} + c \quad \Big| \ln$$

$$\cancel{\tfrac{1}{2}} \cdot \cancel{2}y \cdot \underbrace{\ln(e)}_{=1} = \frac{1}{2} \cdot \ln\left(x^4 + 4\sqrt{x} + c\right) = y(x)$$

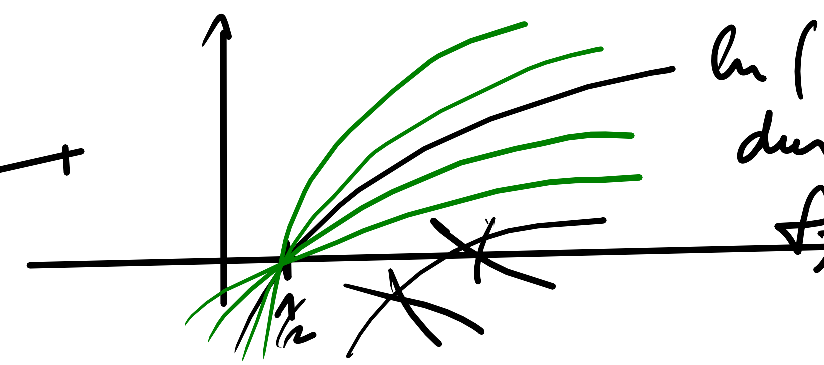
jetzt Anfangswerte

$$0 = \frac{1}{2}\cdot \underbrace{\ln(1+4+c)}_{\text{muss Null sein}} \qquad \text{aber } \ln(1) = 0$$

$$\Rightarrow 1 + 4 + c = 1 \;!$$

$$c = -4$$

$$y(x) = \frac{1}{2}\cdot \ln\left(x^4 + 4\sqrt{x} - 4\right) \;; \quad f(y)$$



AWP zu berechnen: $\dfrac{y'}{y^{3/2}} = \dfrac{x+1}{x\cdot \cancel{y}} \;; \quad y(1) = -\frac{1}{3}$

$$\Rightarrow y' = \frac{dy}{dx} = \frac{x+1}{x}\cdot y^2$$

$$\Rightarrow y' = \left(1 + \frac{1}{x}\right)\cdot y^2 \Rightarrow \int$$

$$\int \frac{1}{y^2}\,dy = \int \left(1 + \frac{1}{x}\right) dx \Longrightarrow -\frac{1}{y} = x + \ln(x) + c$$

$$\Longrightarrow y = \frac{1}{c - x - \ln(x)} \longrightarrow \text{Anfangbed.}$$

mit $x = 1$ $\Longrightarrow -\frac{1}{3} = \frac{1}{c - 1 - 0} \Longrightarrow \underline{\underline{c = -2}}$

$$\Longrightarrow \underline{\underline{y(x) = -\frac{1}{2 + x + \ln(x)}}}$$

AWP: $y' = (x-1)(y-1)$; $y(1) = 0$

$$\frac{1}{y-1}\,dy = (x-1)\,dx \quad \Big|\int$$

$$\ln(y-1) + c_1 = \frac{1}{2}x^2 - x + c_2$$

$$\ln(y-1) = \frac{1}{2}x^2 - x + \underbrace{(c_2 - c_1)}_{c} \quad \Big| e^{\sim\cdots}$$

$$y-1 = e^{\frac{1}{2}x^2 - x + c} \xrightarrow{1.)} \underline{y(x) = e^{\frac{1}{2}x^2 - x + c} + 1}$$

2.) $y = 1 + e^{\frac{1}{2}x^2 - x} \cdot e^{c} \longrightarrow c \quad = c\,e^{\frac{1}{2}x^2 - x} + 1 = y(x)$

Anfangsbed. $y(1) = 0 \Rightarrow c \cdot e^{\frac{1}{2}\cdot 1 - 1} + 1 = 0$

$$c \cdot e^{-\frac{1}{2}} = -1 \Rightarrow \underline{\underline{c = -\sqrt{e}}}$$

AWP: $y' = e^{2y} \cdot \cos(3x) + e^{2y} \cdot \sin\left(x - \frac{\pi}{2}\right)$; $y(0) = -\frac{1}{2}$

$$y' = e^{2y}\left(\cos(3x) + \sin\left(x - \frac{\pi}{2}\right)\right)$$

$$\frac{1}{e^{2y}}\, dy = \left(\cos(3x) + \sin\left(x - \frac{\pi}{2}\right)\right) dx \quad \Rightarrow \int$$

$$-\frac{1}{2} e^{-2y} = \frac{1}{3}\sin(3x) - \cos\left(x - \frac{\pi}{2}\right) + c \quad \Big|\cdot(-2)\Big|\ \ln$$

$$-2y = \ln\left(2 \cdot \cos\left(x - \frac{\pi}{2}\right) - \frac{2}{3}\sin(3x) + c\right)$$

$$\Rightarrow y = \frac{-\ln\left(2 \cdot \cos\left(x - \frac{\pi}{2}\right) - \frac{2}{3}\sin(3x) + c\right)}{2} \rightarrow y(0) = -\frac{1}{2}$$

$$\frac{-\ln\overbrace{(0 - 0 + c)}^{=1}}{2} = -\frac{1}{2} \Rightarrow c = e$$

$$y' = \frac{x^2 + 2y^2}{2xy}$$ lösen

$$\Longrightarrow y' = \frac{x}{2y} + \frac{y}{x}$$

Substitution: $z(x) = \frac{y}{x} \Rightarrow y = x \cdot z(x) \Longrightarrow \frac{dy}{dx} = \frac{1}{2z(x)} + z(x)$

Prod. Regel

$z'(x)$

$z(x) = z$

$$\frac{dy}{dx} = \cancel{z(x)} + x \cdot \frac{dz(x)}{dx} = \frac{1}{2z(x)} + \cancel{z(x)}$$

T.d.V $\Longrightarrow$

$$2z \cdot dz = \frac{1}{x}\, dx \Rightarrow z^2 = \ln(x) + c$$

rücks. $\Longrightarrow$

$$\left(\frac{y}{x}\right)^2 + c = \ln(x) \Longrightarrow y = x \cdot \sqrt{\ln(x) \pm c}$$