Tutorium BMC 13.5.26

Regel von Hospital

$$\lim_{x \to a} f(x) \Rightarrow \frac{f'(x)}{g'(x)} \Rightarrow \frac{f''(x)}{g''(x)} \Rightarrow \ldots$$

Zähler

Nenner

nicht die Quotientenregel

Berechnen Sie: $\lim_{x \to 0} \frac{\cos(x) - 1}{x^2}$ probiere ohne Hospital

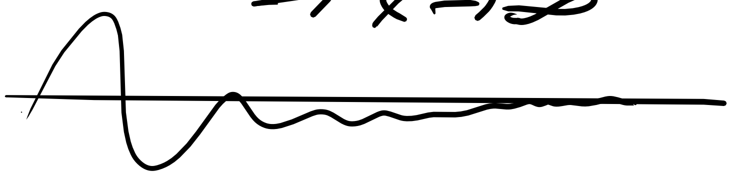
$$\Rightarrow \lim_{x \to 0} \frac{\cos(0) - 1}{0^2} = \frac{0}{0} \text{ ↯} \Rightarrow \lim_{x \to 0} \frac{-\sin(x)}{2x} \xrightarrow{\text{prob.}} \frac{0}{0} \text{ ↯}$$

$$\Rightarrow \lim_{x \to 0} \frac{-\cos(x)}{2} \xrightarrow{\text{prob.}} \frac{-\cos(0)}{2} = \frac{-1}{2} = \underline{\underline{-\frac{1}{2}}}$$

Was wäre wenn $\lim\limits_{x \to \infty} \frac{\cos(x) - 1}{\underbrace{x^2}_{\to \infty}} = \underline{\underline{0}}$

$\frac{1}{x} = \frac{1}{2}, \frac{1}{4}, \frac{1}{100}, \frac{1}{1000}, \dots$

$\to x \to \infty$



$\lim\limits_{x \to 0} \frac{x}{\ln(1+x)} \xrightarrow{\text{prob.}} \frac{0}{\ln(1)} = \frac{0}{0}$ ⚡ $\xRightarrow{\text{Hosp.}}$

$\Rightarrow \frac{1}{\frac{1}{(1+x)}} \Rightarrow \frac{1+x}{1} \xRightarrow[x \to 0]{} \underline{\underline{1}}$

<u>Bruch ist notwendig für</u> Hopital

Was ist bei $x^{-1} \cdot \sin(x)$? ($x \to 0$) $\xrightarrow{\text{erzeugen}}$ $\frac{\sin(x)}{x^1} = \frac{\sin(x)}{x}$

$$\lim_{x\to 3} \frac{x^2-6x+9}{x^2+6x+9} \xrightarrow{\text{prob}} \frac{9-18+9}{9+18+9} = \frac{0}{36} = \underline{\underline{0}}$$

$$\lim_{x\to 0} \frac{x\cdot\sin(x)}{\ln(x^2+1)} \longrightarrow \frac{0\cdot 0}{0} = \text{↯} \xRightarrow{\text{Hop.}} \qquad \boxed{\lim = 1}$$

$$\Longrightarrow \lim_{x\to 0} \frac{\sin(x)+x\cdot\cos(x)}{\left(\frac{1}{x^2+1}\right)\cdot 2x} \overset{H.I}{=} \lim_{x\to 0} \frac{\overset{0}{\sin(x)} + \overset{0}{x\cdot\cos(x)}}{\frac{2x}{(x^2+1)}} = \frac{0}{0} \text{ ↯}$$

$\frac{u}{v}$

H.II.

$$\Longrightarrow \lim_{x\to 0} \frac{\cos(x)+\cos x - x\cdot\sin(x)}{\frac{2\cdot(x^2+1)-2x\cdot 2x}{(x^2+1)^2}} \xrightarrow[x\to 0]{\text{prob.}} \frac{1+1}{2} = \underline{\underline{1}}$$

für Nenner Q.R.

$$\frac{u'v-uv'}{v^2}$$

$$\text{Grenzwert von } f(x) = \lim_{x \to 0} \frac{x \cdot \sin(x)}{x^2 + \sinh^2(3x)} \xrightarrow{\text{prob.}} \frac{0}{0} \ \lightning$$

$$\text{H I} \longrightarrow \lim_{x \to 0} \frac{\sin(x) + x \cdot \cos(x)}{2x + 2 \cdot \sinh(3x)\cosh(3x) \cdot 3} = \frac{\sin(x) + x \cdot \cos(x)}{2x + 6 \cdot \underbrace{\sinh(3x)}_{u} \cdot \underbrace{\cosh(3x)}_{v}}$$

$$\frac{0}{0} \ \lightning$$

$$\text{H II} \quad \lim_{x \to 0} \frac{\overbrace{\cos(x) + \cos(x)}^{2} - x \cdot \sin(x)}{2 + \underbrace{\underbrace{6\cosh(3x) \cdot 3}_{u'} \cdot \underbrace{\cosh(3x)}_{v}}_{} + \underbrace{\underbrace{6 \cdot \sinh(3x)}_{u} \cdot \underbrace{\sinh(3x) \cdot 3}_{v'}}_{0}}$$

$$\frac{2}{2 + 18} = \frac{2}{20} = \frac{1}{10} = \underline{\underline{0{,}1}}$$

Vorsicht

$$\lim_{x\to\infty} \frac{\sinh(x)}{\cosh(x)} \xrightarrow{\text{prob.}} \;\lightning\; \overset{\text{H I.}}{\Longrightarrow} \frac{\cosh(x)}{\sinh(x)} \;\lightning\; \overset{\text{H II}}{\Longrightarrow} \frac{\sinh(x)}{\cosh(x)}$$

$$\lim_{x\to\infty} \frac{\cancel{\tfrac{1}{2}}\left(e^{x}-e^{-x}\right)}{\cancel{\tfrac{1}{2}}\left(e^{x}+e^{-x}\right)} \xrightarrow{\text{HI.}} \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}} = \frac{\cancel{e^{x}}\left(1+e^{-2x}\right)}{\cancel{e^{x}}\left(1-e^{-2x}\right)}$$

$$= \frac{1+\frac{1}{e^{2x}}}{1-\frac{1}{e^{2x}}} \xrightarrow{x\to\infty} \lim = 1$$

$\frac{1}{e^{2x}} \to 0$

$\frac{1}{e^{2x}} \to 0$

Berechnen Sie bitte

$$\lim_{x \to 0} \frac{\sin^2(x)}{\cos(x) - 1} \xrightarrow{HI} \frac{2 \cdot \sin(x) \cdot \cos(x) \cdot 1}{-\sin(x)}$$

$$\underset{x \to 0}{\overset{HII}{\Longrightarrow}} \frac{2 \cdot (\cos(x) \cdot \cos(x) - \sin(x) \cdot \sin(x))}{-\cos(x)} \longrightarrow \frac{2}{-1} = \underline{\underline{-2}}$$

$$\lim_{x \to 1} \frac{(1-x^2)^3}{(1-x^3)^2} \xrightarrow{HI} \frac{\cancel{3}(1-x^2)^2 \cdot (\cancel{-}2x)}{\cancel{2} \cdot (1-x^3)^1 \cdot (\cancel{-}3x^2)} \longrightarrow \frac{(1-x^2)^2}{\underbrace{(1-x^3) \cdot x}_{\overset{1}{=} x - x^4}}$$

$$\underset{x \to 1}{\overset{HII}{\Longrightarrow}} \frac{2(1-x^2) \cdot (-2x)}{1-4x^3} \quad \frac{0}{-3} = \underline{\underline{0}}$$