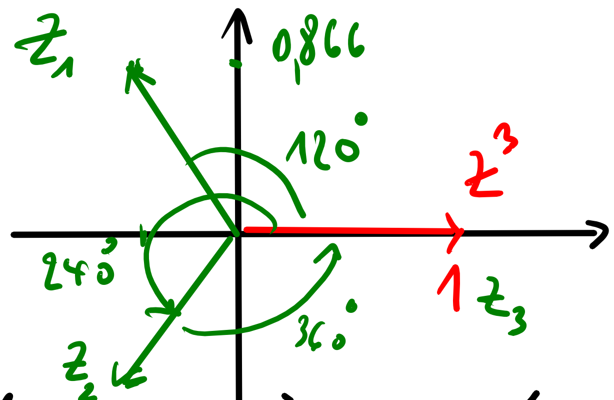
Tutorium Math f. BMC1 18.12.25

Lothar Papula Mathem. f. Ingenieure Band 2

Komplexe Zahlen / komplexe Wurzeln

Berechnen Sie bitte alle Lösungen von: $z^3 = 1$



$$\Rightarrow z = \sqrt[3]{z^3} = \left(z^3\right)^{\frac{1}{3}}$$

$\varphi = 0°$ $|z| = 1$ $\Rightarrow z = \left(1\cdot e^{i\cdot 0}\right)^{\frac{1}{3}} = 1$???

Aber sin und cos sind 2π periodisch

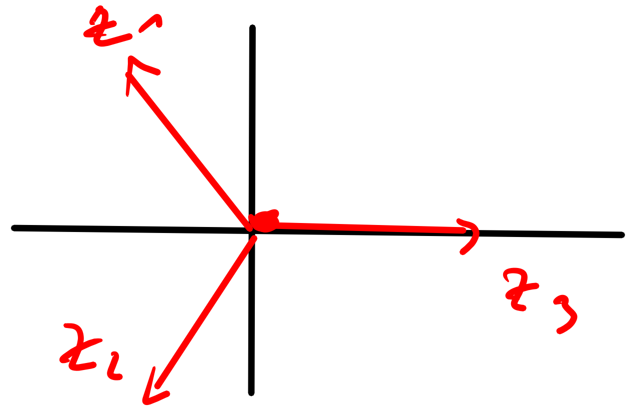
$$z^3 = 1\cdot e^{i\cdot 2\pi} \Rightarrow z_1 = \left(e^{i2\pi}\right)^{\frac{1}{3}} = e^{i\frac{2\pi}{3}} = \cos\frac{2\pi}{3} + i\cdot\sin\frac{2\pi}{3}$$

Probe: $\left(-0,5+i\frac{\sqrt{3}}{2}\right)\left(-0,5+i\frac{\sqrt{3}}{2}\right)\left(-0,5+i\frac{\sqrt{3}}{2}\right) = 0,25 - i\frac{\sqrt{3}}{4} + i\frac{\sqrt{3}}{4} + \frac{3}{4} = 1$

$z_1 = -0,5 + i\frac{\sqrt{3}}{2}$

$$z_2 = e^{i\left(\frac{2\pi}{3}+\frac{2\pi}{3}\right)} = e^{i\frac{4\pi}{3}} = \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right) = \underline{\underline{-0{,}5 - i\frac{\sqrt{3}}{2}}}$$

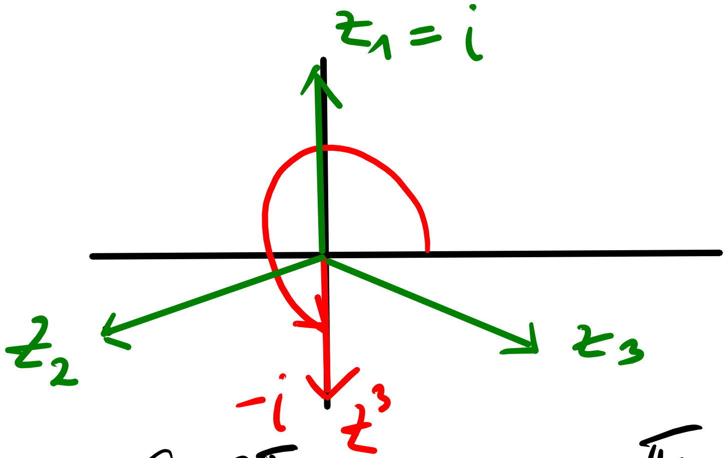
$$z_3 = e^{i\left(\frac{4\pi}{3}+\frac{2\pi}{3}\right)} = e^{i2\pi} = \underline{\underline{1}}$$



$$z^3 = -\left(i^9\right) = -\left(i \cdot i^8\right) = -\left(i\left(i^2\right)^4\right) = \underline{-i = z^3}$$

$$(-1)^4 = 1$$

Alle Lösungen, in welchem/n Quadranten liegt/liegen keine Lösungen?



$z^3 = -i$

$\varphi_{z^3} = 270° = \frac{3\pi}{2}$ $\quad |z^3| = 1$

$z_1 = \left(e^{\frac{3\pi}{2}}\right)^{\frac{1}{3}} = e^{\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$ $\quad \Delta\varphi = 120° = \frac{2\pi}{3}$

$z_2 = \ldots \frac{\pi}{2} + \frac{2\pi}{3} \ldots \Rightarrow \frac{7\pi}{6} \Rightarrow \cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right) = -0{,}866 - 0{,}5i$

$z_3 \Rightarrow \underbrace{330°}_{\hat{=}\ \frac{7\pi}{6} + \frac{2\pi}{3}} \Rightarrow 0{,}866 - 0{,}5i$

Antwort: in I. und II. liegen keine Lösungen

Schreiben Sie in exponentieller Form: $\sin(30°) = \cos(90° - 30°)$

a) $z = 1\sin\left(\frac{\pi}{3}\right) + i \cdot 1\cos\left(\frac{\pi}{3}\right)$

trigon. Regeln $\quad \underset{90-\varphi}{\cos\left(\frac{\pi}{2} - \frac{\pi}{3}\right)} \qquad \underset{90-\varphi}{\sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right)}$

$\Rightarrow \cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)$

$\Rightarrow \cdot\ e^{i\frac{\pi}{6}}$

natürl. gilt auch $\sin(\varphi) = \sin\left(\frac{\pi}{2} + \varphi\right) \Rightarrow \sin(30°) = \sin(120°)$

aber $\cos(30) \neq \cos(120)$

weil beide im 1. Q pos. sind.

b) Geben Sie alle $x \in \mathbb{R}$ an für die $z = \frac{1}{(2+ix)^3}$ reell ist

Berechne: $(2+ix)(2+ix) = 4+2ix+2ix-x^2$

$$(4+4ix-x^2)(2+ix) = 8+4ix+8ix-4x^2-2x^2-ix^3$$

$$\Rightarrow 8+\underline{12ix}-6x^2\underline{-ix^3} \Rightarrow 12ix-ix^3=0$$

$$\Rightarrow 12\cancel{i}\cancel{x} = \cancel{i}\cancel{x}^{3/2} \Rightarrow x^2 = 12 \Rightarrow \underline{\underline{x = \pm\sqrt{12}}}$$

!! x kürzen = Lösung fällt weg

<u>Probe:</u> $(2+\sqrt{12}\,i)(2+\sqrt{12}\,i) = (4+2\sqrt{12}\,i+2\sqrt{12}\,i-12)\cdot(2+\sqrt{12}\,i)$

$$\Rightarrow -16-8\sqrt{12}\,i+8\sqrt{12}\,i-48 = -64 \in \mathbb{R}$$

$$\Rightarrow z = \frac{-1}{64} \in \mathbb{R}$$

nicht vergessen $x=0$ auch

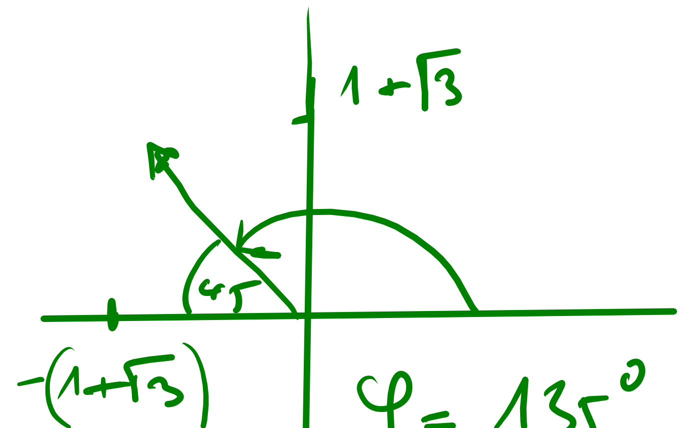
Bestimmen Sie bitte den Imaginärteil und das Argument (φ) von $z = 2\cdot e^{i\frac{5}{6}\pi} - \dfrac{4}{1+i\sqrt{3}}$

$$\underbrace{2\cdot\left(\cos\left(\tfrac{5}{6}\pi\right) + i\sin\left(\tfrac{5\pi}{6}\right)\right)} - \frac{4(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})}$$

$$= -\sqrt{3} + i - \frac{4-4\sqrt{3}\,i}{1+3} = -\sqrt{3} + i - (1 - i\sqrt{3})$$

$$= -\sqrt{3} + i - 1 + \sqrt{3}\,i \implies i\underbrace{(1+\sqrt{3})}$$

$$\implies -1 - \sqrt{3} = -(1+\sqrt{3}) \qquad \mathrm{Im}(z) = 1 + \sqrt{3}$$



$\varphi = 135^\circ$