Tutorium Math. BMC 2 27.5.26

Kurze Beispiele zur Festigung

1. Integral bei abhängigen Grenzen
2. Integral bei Rest

Bei festen Grenzen $\longrightarrow$ Reihenfolge egal

$$\iint dx\,dy \qquad 1 \leq x \leq 2 \qquad 0 \leq y \leq 1$$

$$\int_1^2 \int_0^1 1 \cdot dx\,dy = \int_1^2 dx \int_0^1 dy = x\Big]_1^2 \cdot y\Big]_0^1 = \underline{\underline{1}}$$

$$\int_0^1 \int_1^2 1\,dy\,dx = \int_0^1 dy \int_1^2 dx = y\Big]_0^1 \cdot x\Big]_1^2 = \underline{\underline{1}}$$

$\iint 1 \cdot dy\, dx \quad ; \quad 1 \le x \le 2 \quad \underbrace{0 \le y \le x}$ 1.) V.E.

$$\int_1^2 \int_0^x dy\, dx = \int_1^2 dx \int_0^x dy = \int_1^2 dx \cdot y \Big]_0^x = \int_1^2 x\, dx = \frac{1}{2} x^2 \Big]_1^2 = \underline{\underline{\frac{3}{2}}}$$

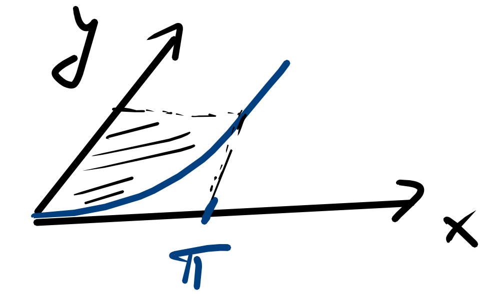
Jetzt falsch angefangen

$$\int_1^2 \int_0^x 1 \cdot dx\, dy \Rightarrow \int_1^2 dy \int_0^x dx \Rightarrow \int_1^2 dy\; x \Big]_0^x = \int_1^2 x\, dy = x \cdot y \Big]_1^2$$

$$= 2x - x = \underline{\underline{x}}$$

<u>Weitere Beispiele:</u>

$$f(x,y) = \sin\left(\frac{y}{x}\right) \qquad A = \left\{(x,y) \,/\, 0 < x \le \pi \,;\, 0 \le y \le x^2\right\}$$



$$\int_0^{\pi} dx \cdot \int_0^{x^2} \sin\left(\frac{y}{x}\right) dy = \int_0^{\pi} dx \left[-x \cdot \cos\left(\frac{y}{x}\right)\right]_0^{x^2}$$

$$= \int_0^{\pi} dx \cdot \left[\underbrace{-x \cdot \cos(x)}_{\text{Tabelle}} + x\right] \cdot dx$$

$$\Rightarrow \left[-x \cdot \cancel{\sin}(x) - \cos(x)\right]_0^{\pi} + \left[\frac{1}{2}x^2\right]_0^{\pi} \Rightarrow 2 + \frac{\pi^2}{2} \approx 6{,}935$$

$$--1 --1 = 2$$

V. E.

$$f(x,y) = \frac{x \cdot \sinh(x \cdot y)}{\cosh(x)} \qquad A = \ldots \quad 0 \le x \le \frac{1}{2}\ln(3) \;;\; \underbrace{0 \le y \le \frac{\operatorname{arcosh}(2)}{x}}_{\mathrm{I.}}$$

$$\Rightarrow \int_0^{\frac{1}{2}\ln(3)} dx \int_0^{\frac{\operatorname{arcosh}(2)}{x}} \frac{x \cdot \sinh(x \cdot y)}{\cosh(x)}\, dy$$

$$\Rightarrow \int_0^{\frac{1}{2}\ln(3)} \frac{x}{\cosh(x)}\, dx \int_0^{\frac{\operatorname{arcosh}(2)}{x}} \sinh(x \cdot y)\, dy$$

$$\left[\frac{1}{x} \cdot \cosh(\cancel{x} \cdot y) \right]_0^{\frac{\operatorname{arcosh}(2)}{\cancel{x}}} \rightarrow \frac{2}{x}$$

$\operatorname{arcosh}(2) \longrightarrow$ ergibt 2

$$\nearrow \ln\left(3^{-\frac{1}{2}}\right)$$

$$-\frac{1}{2}\ln(3) = -\ln 3^{\frac{1}{2}} - \ln\sqrt{3}$$

$$\Rightarrow \int_0^{\frac{1}{2}\ln(3)} \frac{\cancel{x}}{\cosh(x)} \cdot \frac{2}{\cancel{x}}\, dx \Rightarrow 2 \cdot \int_0^{\frac{1}{2}\ln(3)} \underbrace{\frac{1}{\cosh(x)}}_{\text{Tabelle}}\, dx$$

$$-4\left(\arctan\left(e^{-x}\right)\right]_{0}^{\frac{\ln(3)}{2}} \Rightarrow -4\left(\arctan\left(e^{-\frac{\ln(3)}{2}}\right) - \arctan\left(e^{-0}\right)\right)$$

$$-4\cdot\left(\underbrace{\arctan\left(\frac{1}{\sqrt{3}}\right)}_{\frac{\pi}{6}} - \underbrace{\arctan(1)}_{\frac{\pi}{4}}\right) = \frac{\pi}{3} \text{ V.E.}$$

$$f(x,y) = x\cdot\sqrt{x}\cdot\cos\left(x^2 - y\sqrt{x}\right) \;;\; \underbrace{1 \le x \le 2}_{\mathrm{I}.} \;;\; \underbrace{\frac{1}{\sqrt{x}} \le y \le x\cdot\sqrt{x}}_{\to\,\mathrm{I}.}$$

$$\int_{1}^{2} x\cdot\sqrt{x}\,dx \int_{\frac{1}{\sqrt{x}}}^{x\cdot\sqrt{x}} \cos\left(x^2 - y\sqrt{x}\right)\,dy = \int_{1}^{2} x\cdot\sqrt{x}\,dx \cdot \left[\sin\left(x^2 - y\sqrt{x}\right)\cdot\left(-\frac{1}{\sqrt{x}}\right)\right]_{\frac{1}{\sqrt{x}}}^{x\sqrt{x}}$$

$$\int_1^2 -x \cdot \left(\cancel{\sin\left(x^2 - \underbrace{x \cdot \sqrt{x}\sqrt{x}}_{x^2}\right)}_{=0} - \sin\left(x^2 - \frac{1}{\sqrt{x}} \cdot \sqrt{x}\right)\right)$$

$$- \sin\left(x^2 - 1\right)$$

$$\Rightarrow \int_1^2 x \sin\left(x^2 - 1\right) \Rightarrow \left[-\frac{\cos\left(x^2 - 1\right)}{2} \right]_1^2 = -\frac{\cos(3)}{2} - -\frac{\cos(0)}{2}$$

$$= \underline{\frac{1}{2}\left(1 - \cos(3)\right)}$$