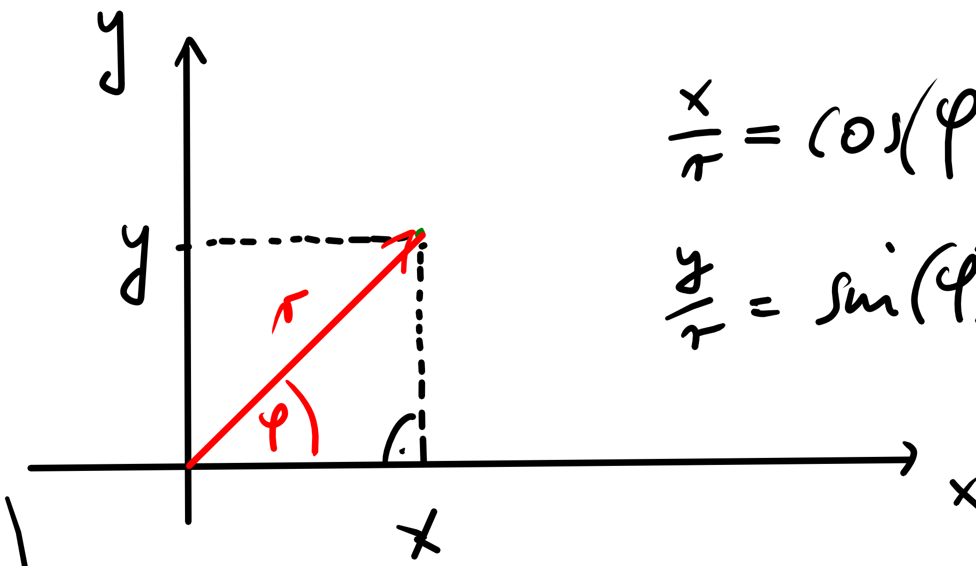
Tutorium Math. BMC2 3.6.26

Polarkoordinaten



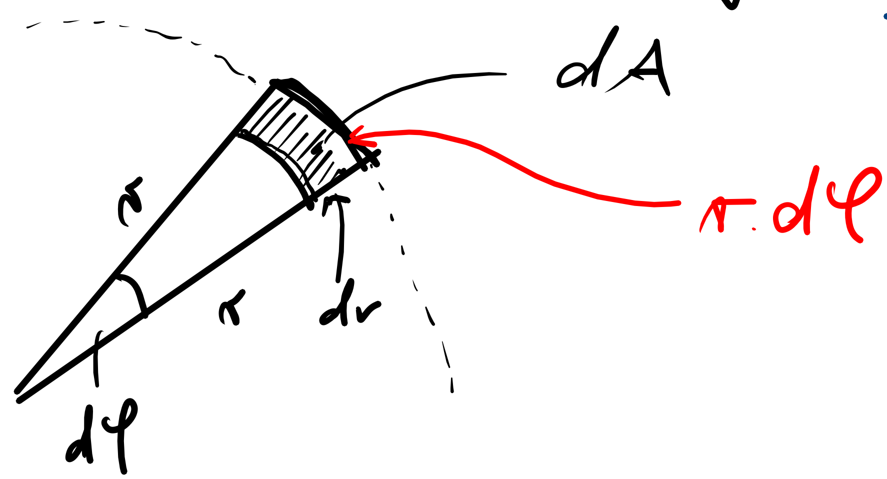
$$\frac{x}{r} = \cos(\varphi) \Rightarrow \boxed{x = r\cdot\cos(\varphi)}$$

$$\frac{y}{r} = \sin(\varphi) \Rightarrow \boxed{y = r\cdot\sin(\varphi)}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow r\cdot\begin{pmatrix} \cos(\varphi) \\ \sin(\varphi) \end{pmatrix}$$

Kreisgleichung: $\boxed{x^2 + y^2 = r^2}$

Kartesisch: $dA = dx\cdot dy$



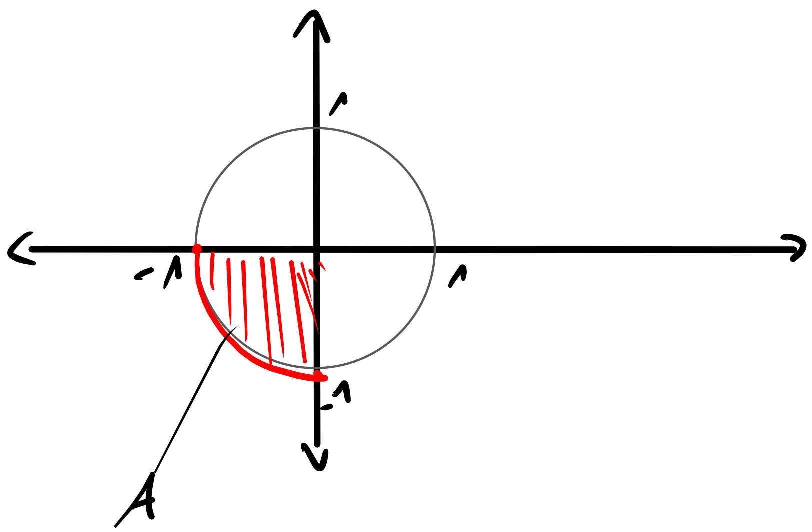
polar:

$$\Rightarrow dA = dr\cdot r\cdot d\varphi = \underline{r\,dr\cdot d\varphi}$$

φ: Bogenmaß

Berechnen Sie: $\displaystyle\iint e^{-(\underbrace{x^2+y^2}_{\hat{=}\, r^2})} \overbrace{dx\,dy}^{dA}$ $\qquad A: x^2+y^2 \leq 1$

$\qquad\qquad x, y \leq 0$



$$\Rightarrow \int_{\pi}^{\frac{3\pi}{2}} d\varphi \int_0^1 \underbrace{r \cdot e^{-r^2}}_{\text{Tafel}} dr \qquad \text{Tafel} \longleftrightarrow \frac{-e^{-r^2}}{2}$$

$$\uparrow$$

$$\varphi\Big]_{\pi}^{\frac{3\pi}{2}} \cdot \frac{-e^{-r^2}}{2}\Big]_0^1 \approx 0{,}496$$

Gegeben sind für $-\frac{\pi}{2} < x < \frac{\pi}{2}$ die DGL:

(h) $\cos(x) \cdot y' + \sin(x) \cdot y = 0$ und $\cos(x) \cdot y' + \sin(x) \cdot y = 1$ (S)

Tipp: $\int \frac{1}{\cos^2(x)}\, dx = \tan(x) + C$

a) Bestimmen Sie jeweils die allg. Lösung d. DGL.

b) Bestimmen Sie die spezielle Lösung $y_s(x)$ mit $y_s(0) = 1$

(h) $\cos(x) \cdot y' + \sin(x) \cdot y = 0 \quad | : \cos(x)$

$\Rightarrow y' + \tan(x) \cdot y = 0 \quad \Rightarrow \frac{dy}{dx} = -\tan(x) \cdot y \quad \Rightarrow \int \frac{1}{y}\, dy = \int -\tan(x)\, dx$

$y' \updownarrow \frac{dy}{dx}$

Tabelle

$$\Rightarrow \ln y = \ln|\cos(x)| + c \quad \Big| \, e^{\sim} \quad \Rightarrow e^{\ln y} = e^{\ln|\cos(x)| + c}$$

$$\Rightarrow y = e^{\ln|\cos(x)|} \cdot \underbrace{e^{c}}_{K} = \underline{\underline{K \cdot |\cos(x)|}} \quad (\hat{=} \, y_h)$$

b) $\cos(x) \cdot y' + \sin(x) \cdot y = 1 \quad \Big| : \cos(x) \quad \Rightarrow y' + \underbrace{\tan(x)}_{a(x)} \cdot y = \underbrace{\frac{1}{\cos(x)}}_{g(x)}$

$$\Rightarrow A(x) = \int a(x) = -\ln|\cos(x)| \qquad \underline{e^{-A(x)}} = e^{--\ln|\cos(x)|} = \underline{\cos(x)}$$

Formel

$$y(x) = \left(k + \int e^{A(x)} \cdot g(x) \cdot dx\right) e^{-A(x)}$$

$$\Rightarrow \left(k + \int \frac{1}{\cos(x)} \cdot \frac{1}{\cos(x)} \, dx\right) \cdot \cos(x)$$

$\underline{k + c = k}$ Tipp beachten

$$\Rightarrow \left(k + \tan(x)\right) \cdot \cancel{\cos(x)}$$

$$\tan(x) = \frac{\sin(x)}{\cancel{\cos(x)}}$$

$$\Rightarrow y(x) = \frac{k}{\cos(x)} + \sin(x) \quad \Rightarrow 1 = \frac{k}{\cos(0)} + \sin(0)$$

$$\underline{\underline{k = 1}}$$

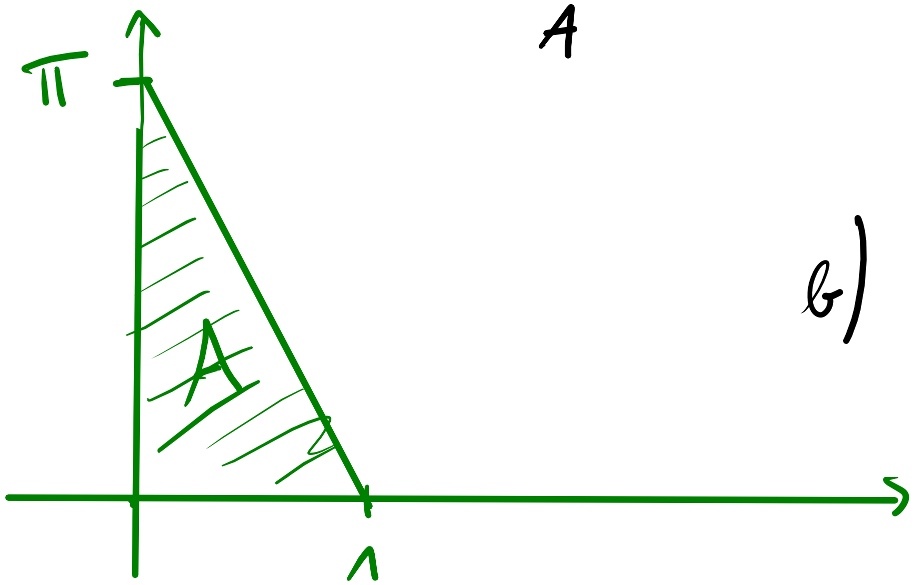
a) Gegeben ist die Menge $A = \Big\{(x,y) \,/\, 0 \le y \overset{\hat{=}\ -\pi\cdot x + \pi}{\le \pi - \pi\cdot x} \quad 0 \le x \le 1\Big\}$

I. II.

Skizzieren Sie A in der x-y Ebene

b) Berechnen Sie $\iint\limits_A \pi\cdot x\cdot\cos(y)\,dA$

a)



b) $\pi\cdot\int_0^1 x\,dx \int_0^{\pi-\pi\cdot x} \cos(y)\,dy = \pi\cdot\int_0^1 x\cdot\big[\sin(x)\big]_0^{\pi-\pi\cdot x}\,dx$

$= \pi\cdot\int_0^1 x\cdot\sin(\pi-\pi\cdot x)\,dx$

Substit

$t = \pi - \pi\cdot x$

$\frac{dt}{dx} = -\pi$

$= \pi\cdot\int_0^{\pi} x\cdot\sin(t)\,\frac{dt}{-\pi}$

$\pi\cdot x = \pi - t \Rightarrow x = \frac{\pi - t}{\pi}$

$= \pi\int_0^{\pi} \frac{\pi - t}{\pi}\cdot -\pi \cdot\sin(t)\,dt$

... nächstes Mal sauber!

$$= \frac{1}{\pi}\int_0^{\pi}(\pi - t)\cdot \sin t \, dt \qquad = \int_0^{\pi} \sin(t)\, dt - \frac{1}{\pi}\cdot\int_0^{\pi} \underbrace{t\cdot\sin(t)}_{\text{Tabelle}}\, dt$$

$$= \left[\cos t\right]_{\pi}^{0} - \frac{1}{\pi}\left[\sin(t) - t\cdot\cos(t)\right]_0^{\pi}$$

$$= 2 - 1 = \underline{\underline{1}} \text{ U.E.}$$