BMC 3.6.26

$$\int_{e^{\pi/4}}^{e^{\pi/2}} \frac{\sin(\ln(x^2))}{x}\,dx \qquad z = \ln(x^2) \Rightarrow \frac{dz}{dx} = \frac{1}{x^2}\cdot 2x = \frac{2}{x}$$

$$\Rightarrow dx = \frac{x}{2}\,dz$$

$$x_1 = e^{\pi/4} \Rightarrow \underline{\underline{z_1}} = \ln\left(e^{\pi/4}\right)^2 = \ln\left(e^{\pi/2}\right) = \underline{\underline{\frac{\pi}{2}}}$$

$$x_2 = e^{\pi/2} \Rightarrow z_2 = \ln\left(e^{\pi}\right) = \underline{\underline{\pi}}$$

$$\Rightarrow \int_{\pi/2}^{\pi} \frac{\sin(z)}{\cancel{x}}\cdot\frac{\cancel{x}}{2}\,dz = \frac{1}{2}\int_{\pi/2}^{\pi} \sin(z)\,dz = \frac{1}{2}\Big[-\cos(z)\Big]_{\pi/2}^{\pi}$$

$$= \frac{1}{2}\Big[\underbrace{-\cos(\pi)}_{=-1} - \cancel{-\cos\left(\frac{\pi}{2}\right)}_{=0}\Big] = \underline{\underline{\frac{1}{2}}}$$

Bestimmen Sie den Wert des bestimmten Integrals

$$I = \int_0^{\pi/3} \underbrace{x}_{u} \cdot \underbrace{\cos\left(\frac{\pi}{2}-x\right)}_{v'} \, dx$$

$$\boxed{\begin{array}{l}\text{part. Integr.} \\ \int u v' = u \cdot v - \int u' v\end{array}}$$

$u = x \qquad u' = 1$

$v = -\sin\left(\frac{\pi}{2}-x\right) \qquad v' = \cos\left(\frac{\pi}{2}-x\right)$

$$\Rightarrow \int u v' = -x \cdot \sin\left(\frac{\pi}{2}-x\right)\Big]_0^{\pi/3} - \int_0^{\pi/3} -\sin\left(\frac{\pi}{2}-x\right) dx$$

$$= \underbrace{-\frac{\pi}{3} \sin\Big(\overbrace{\frac{\pi}{2}-\frac{\pi}{3}}^{\pi/6 \,\hat{=}\, 30^\circ}\Big)}_{-\frac{\pi}{3} \cdot \frac{1}{2}} - \cancel{-0} + \int_0^{\pi/3} \sin\left(\frac{\pi}{2}-x\right) dx$$

$$-\frac{\pi}{3} \cdot \frac{1}{2} = -\frac{\pi}{6} + \left[\cos\left(\frac{\pi}{2}-x\right)\right]_0^{\pi/3}$$

$+\cos\left(\frac{\pi}{2}-x\right)$

$$\Rightarrow -\frac{\pi}{6} + \Big[\underbrace{\cos\Big(\overbrace{\frac{\pi}{2}-\frac{\pi}{3}}^{\pi/6}\Big)}_{\sqrt{3}/2} - \cancel{\cos\left(\frac{\pi}{2}-0\right)}\Big]$$

$$I = -\frac{\pi}{6} + \frac{\sqrt{3}}{2}$$

Bestimmen Sie eine Stammfunktion von:

$$\int x^n = \frac{1}{n+1} \cdot x^{n+1}$$

$$f(x) = 20x^4 + \frac{1}{\sqrt[4]{x^5}} + (7+32x)^{31}$$

$$20 \cdot \frac{1}{5} x^5 = \underline{\underline{4 \cdot x^5}}$$

$$x^{-\frac{5}{4}}$$

$$\frac{1}{-\frac{5}{4}+1} \cdot x^{-\frac{5}{4}+1}$$

$$\frac{1}{-\frac{1}{4}} x^{-\frac{1}{4}}$$

$$\underline{\underline{-4 / \sqrt[4]{x}}}$$

$$\frac{1}{32}(7+32x)^{32} \cdot \frac{1}{32} = \underline{\underline{\frac{1}{32^2}(7+32x)}}$$

$$f(x) = 6x^{13} + 7\cdot\sqrt[3]{x^4} + \frac{7}{(2-x)^{15}} + \cos(2x)$$ (Stammf.)

$$\int 6\cdot x^{13} = 6\cdot\frac{1}{14}\cdot x^{14} = \frac{3}{7}x^{14}$$

$$\int 7\cdot\sqrt[3]{x^4} = \int 7\, x^{\frac{4}{3}} = 7\,\frac{1}{\frac{7}{3}}\,x^{\frac{7}{3}} = 7\cdot\frac{3}{7}\cdot x^{\frac{7}{3}} = 3\sqrt[3]{x^7}$$

$$\int \frac{7}{(2-x)^{15}} = \int 7\cdot(2-x)^{-15} = 7\cdot\frac{-1}{-14}\cdot(2-x)^{-14} = \frac{7}{14}\cdot(2-x)^{-14} = \frac{1}{2}(2-x)^{-14}$$

$$= \frac{1}{2(2-x)^{14}}$$

$$\int \cos(2x) \Longrightarrow \sin(2x)\cdot\frac{1}{2} = \underline{\underline{\frac{1}{2}\cdot\sin(2x)}}$$

Stammfunktion von $f(x) = \frac{1+4x}{2} + (2-x)^{20} + \frac{3}{x^8} + 7\sqrt[4]{x^3}$

$$\int \frac{1+4x}{2} = \frac{1}{2}\int 1+4x = \frac{1}{2}\left[\int 1\,dx + \int 4x\,dx\right] = \frac{1}{2}\left[x + 4\cdot\frac{1}{2}x^2\right] = \frac{1}{2}x + \frac{1}{2}\cdot 4\cdot\frac{1}{2}x^2$$

$$= \frac{1}{2}x + x^2$$

$$\int (2-x)^{20} = \frac{1}{21}\cdot(2-x)^{21}\cdot\left(\frac{1}{-1}\right) = -\frac{1}{21}(2-x)^{21}$$

$+C$

$$\int \frac{3}{x^8} = \int 3\cdot x^{-8} = 3\cdot\frac{1}{-7}x^{-7} = -\frac{3}{7}\cdot\frac{1}{x^7}$$

$$\int 7\cdot\sqrt[4]{x^3} = \int 7\cdot x^{\frac{3}{4}} = 7\cdot\frac{1}{\frac{7}{4}}x^{\frac{7}{4}} = \cancel{7}\cdot\frac{4}{\cancel{7}}\cdot x^{\frac{7}{4}} = 4\sqrt[4]{x^7} + C$$