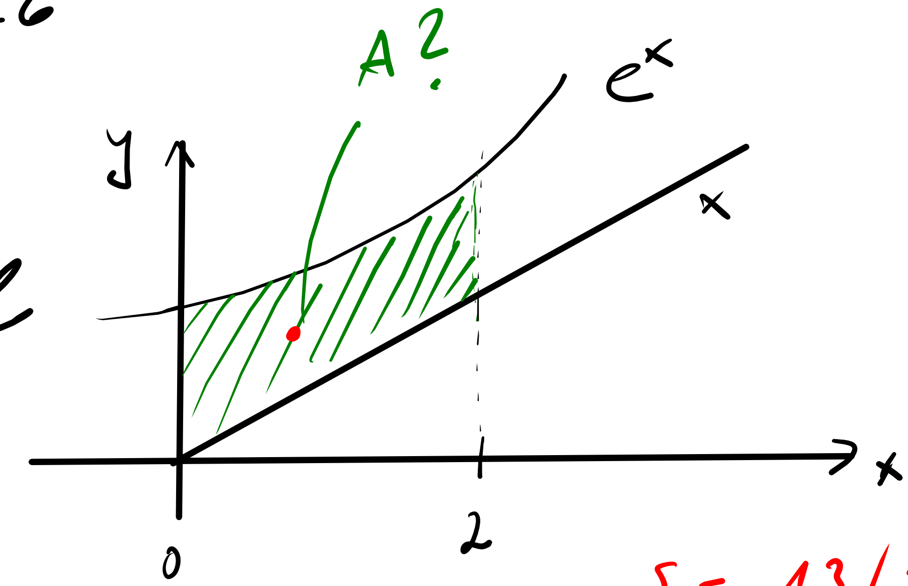
Tutorium Math BMC2 10.6.26

Doppelintegrale polar



S = 1,3 / 2,75

Noch ein Kart. Beispiel: Doppelintegral Flächenberechnung.

$$A = \iint dA \Rightarrow \int_0^2 dx \int_x^{e^x} dy = \int_0^2 dx \, y\Big|_x^{e^x}$$

$$= \int_0^2 (e^x - x)\,dx = e^x\Big|_0^2 - \frac{1}{2}x^2\Big|_0^2 = e^2 - 1 - 2 = e^2 - 3 \approx 4{,}39 \text{ F.E.}$$

Berechnen Sie $\iint \sqrt{x^2+y^2}\,dx \cdot dy$ in Polarkoord.

$0 \le r \le 4$ I.

$0 \le \varphi \le \frac{\pi}{2}$ II.

$$\int_0^{\pi/2} d\varphi \int_0^{\varphi} \underbrace{r \cdot}_{r^2} r\,dr$$

$$r^2 = x^2 + y^2 \Rightarrow r = \sqrt{x^2+y^2}$$

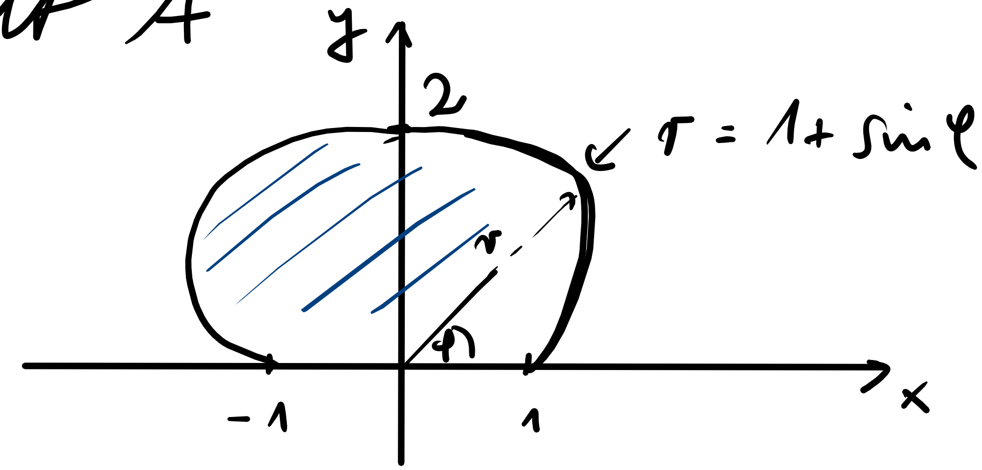
$$dA = r\,dr \cdot d\varphi$$

$$= \int_0^{\pi/2} d\varphi \cdot \frac{1}{3} r^3 \Big]_0^{\varphi} = \int_0^{\pi/2} d\varphi \cdot \frac{1}{3}\varphi^3 = \frac{1}{3}\cdot\frac{1}{4}\varphi^4 \Big]_0^{\pi/2} = \frac{1}{3}\cdot\frac{1}{4}\cdot\left(\frac{\pi}{2}\right)^4$$

$$= \frac{1}{3}\cdot\frac{1}{4}\cdot\frac{1}{16}\cdot\pi^4 = \frac{\pi^4}{192}$$

Berechnen Sie den Flächeninhalt A

$r = 1 + \sin\varphi \qquad 0 \le \varphi \le \pi$



$$A = \iint dA = \int_0^{\pi} d\varphi \int_0^{1+\sin\varphi} r\, dr$$

Int. nach r $\Longrightarrow$ $\frac{1}{2} r^2 \Big]_0^{1+\sin\varphi} = \frac{1}{2}(1+\sin\varphi)^2 = \frac{1}{2}\left(1+2\sin\varphi+\sin^2\varphi\right)$

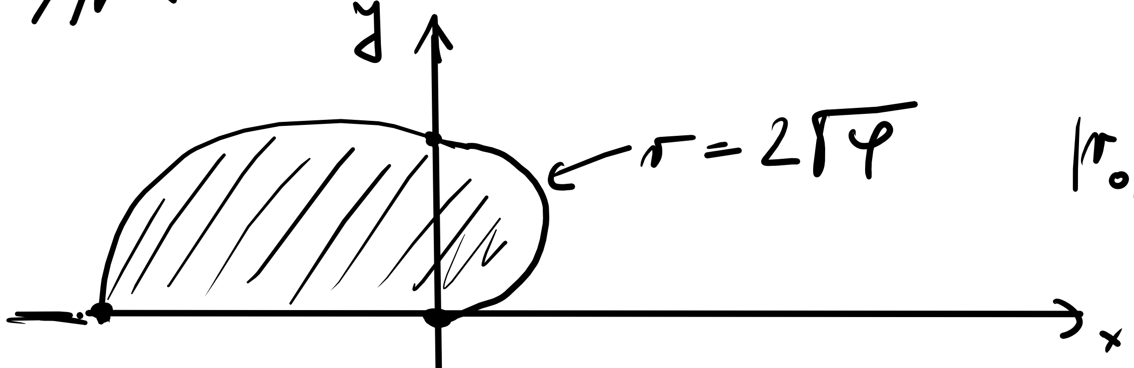
Int. nach φ $\Longrightarrow$ $A = \frac{1}{2}\int_0^{\pi}\left(1+2\sin\varphi+\underbrace{\sin^2\varphi}_{\text{Tab. 205; } a=1}\right)d\varphi$

$$= \frac{1}{2}\left[\varphi - 2\cos(\varphi) + \frac{\varphi}{2} - \frac{\sin(2\varphi)}{4}\right]_0^{\pi}$$

$$= \frac{1}{2}\left[\frac{3}{2}\varphi - 2\cos(\varphi) - \frac{1}{4}\cdot\sin(2\varphi)\right]_0^{\pi} =$$

$$= \frac{1}{2}\left(\frac{3}{2}\pi + 2 + 2\right) = \frac{1}{2}\left(\frac{3}{2}\pi + 4\right) = \underline{\underline{\frac{3}{4}\pi + 2}} \approx 4{,}36$$

Die in Polarkoord. def. Kurve $r = 2\sqrt{\varphi}$, $0 \le \varphi \le \pi$ bildet mit der x-Achse ein Flächenstück der Fläche A die zu bestimmen ist.

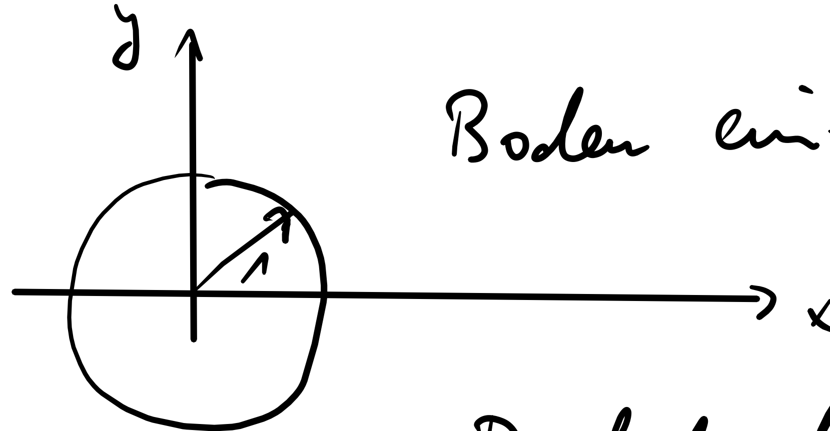


$|r_0| = 0 \quad |r_\pi| = 2\sqrt{\pi} \approx 3{,}54$

$$\int_0^{\pi} d\varphi \int_0^{2\sqrt{\varphi}} r\,dr$$

I. $\frac{1}{2} r^2 \Big]_0^{2\sqrt{\varphi}} = \frac{1}{2} \cdot 4\varphi = \underline{\underline{2\varphi}}$

II. $\int_0^{\pi} 2\varphi\,d\varphi = \varphi^2 \Big]_0^{\pi} = \underline{\underline{\pi^2}}$



Boden eines Zylinders $0 \le r \le 1$

$0 \le \varphi \le 2\pi$

Deckel des Zylinders $z = e^{x^2+y^2}$

$z = e^{r^2}$

Berechnen Sie das Zylindervolumen (Substitution nötig)

Polarkoord.

$$V = \int_0^{2\pi} d\varphi \int_0^1 e^{r^2} \cdot r \cdot dr$$

mit $u = r^2$ $\quad \frac{du}{dr} = 2r \quad dr = \frac{du}{2r}$

Grenzen: unten $r = 0$ $u = 0$

oben $r = 1$ $u = 1$

$$\Rightarrow \int_0^1 e^u \cdot \cancel{r} \cdot \frac{du}{2\cancel{r}} = \frac{1}{2}\int_0^1 e^u \, du = \frac{1}{2} e^u \Big]_0^1 = \frac{1}{2}(e^1 - e^0) = \underline{\underline{\frac{1}{2}(e-1)}}$$

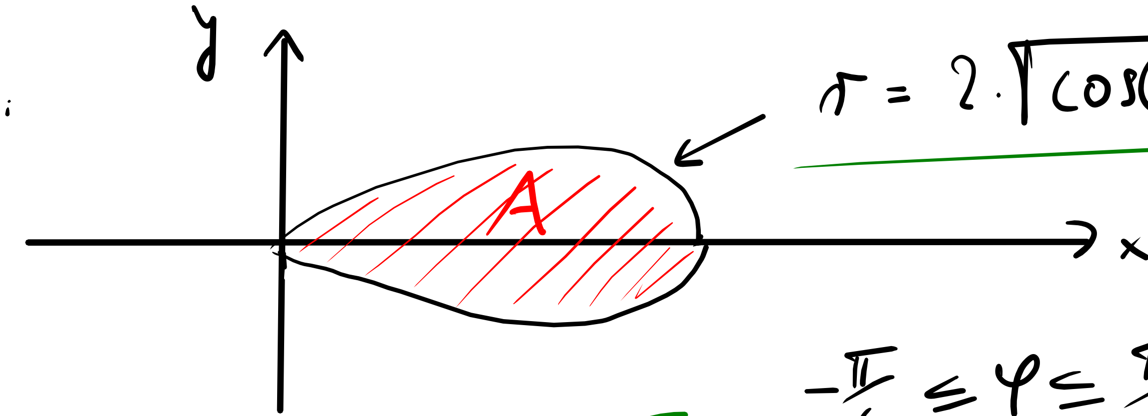
$$V \Rightarrow \frac{1}{2}(e-1)\cdot\int_0^{2\pi} 1\, d\varphi = \frac{1}{2}(e-1)\cdot \varphi\Big]_0^{2\pi} = \frac{1}{2}(e-1)\cdot 2\pi = \pi(e-1)$$

$\approx 5{,}4$ V.E.

Flächenberechnung:

A gesucht

nächstes mal fertig



$r = 2\cdot\sqrt{\cos(3\varphi)}$ I.

II. $-\frac{\pi}{6} \le \varphi \le \frac{\pi}{6}$

$\frac{\pi}{6} \,\hat{=}\, 30°$