Math. BMC 10.6.26

Berechnen Sie mit geeigneter Substitution

$$\int e^x \cdot \sin(e^x)\, dx \qquad u = e^x \qquad \frac{du}{dx} = e^x \Rightarrow dx = \frac{du}{e^x}$$

$$\rightarrow \int \cancel{u} \cdot \sin(u) \frac{du}{\cancel{e^x}} = \int \sin u \, du = -\cos(u) + c$$

$$\Rightarrow \underline{\underline{-\cos(e^x) + c}}$$

mit Subst.

$$\int_1^{\cancel{e^\pi}} \frac{\sin(\ln(x))}{x}\, dx \qquad u = \ln(x)$$

$u = \ln(x) \qquad \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x \cdot du$

Grenzen $\quad x_1 = 1 \Rightarrow \underline{\underline{u_1}} = \ln(x_1) = \ln(1) \underline{\underline{= 0}}$

$x_2 = e^{\pi} \Rightarrow \underline{\underline{u_2}} = \ln(e^{\pi}) \underline{\underline{= \pi}}$

$$\int_{u_1=0}^{u_2=\pi} \frac{\sin(u)}{\cancel{x}} \cdot \cancel{x} \cdot du$$

$$= \int_0^{\pi} \sin(u)\, du = -\cos(u)\Big|_0^{\pi} = \underbrace{-\cos(\pi)}_{=-1} - \underbrace{-\cos(0)}_{=-1} \underline{\underline{= 2}}$$

Bestimmen Sie

$$\lim_{x \to 0} \sqrt{x \cdot \tan\left(\frac{\pi}{2} - x\right)} \xrightarrow{\text{einsetzen}} \sqrt{0 \cdot \infty} \; ↯ \Rightarrow \text{Hospital}$$

$$\Rightarrow \sqrt{x}\cdot\sqrt{\tan\left(\frac{\pi}{2}-x\right)} = x^{\frac{1}{2}}\cdot\left(\tan\left(\frac{\pi}{2}-x\right)\right)^{\frac{1}{2}}$$

$$= \frac{\left(\tan\left(\frac{\pi}{2}-x\right)\right)^{\frac{1}{2}}}{x^{-\frac{1}{2}}} \xrightarrow[x\to 0]{H.} \frac{\frac{1}{2}\left(\tan\left(\frac{\pi}{2}-x\right)\right)^{-\frac{1}{2}}\cdot\frac{1}{\cos^2\left(\frac{\pi}{2}-x\right)}\cdot(-1)}{-\frac{1}{2}\cdot x^{-\frac{3}{2}}}$$

Irrweg!

Bruch anders herum

$$\frac{x^{\frac{1}{2}}}{\tan\left(\frac{\pi}{2}-x\right)^{-\frac{1}{2}}} \overset{H.}{\Longrightarrow} \frac{\cancel{\frac{1}{2}}\,x^{-\frac{1}{2}}}{+\cancel{\frac{1}{2}}\left(\tan\left(\frac{\pi}{2}-x\right)\right)^{-\frac{3}{2}}\cdot\frac{1}{\cos^2\left(\frac{\pi}{2}-x\right)}\cdot(\cancel{+}1)}$$

$$\frac{x^{-\frac{1}{2}} \cdot \cos^2\left(\frac{\pi}{2}-x\right)}{\left(\tan\left(\frac{\pi}{2}-x\right)\right)^{-3/2}} \quad ?$$

$$\int_{\pi/6}^{\pi/2} \frac{\ln(\sin(x))}{\tan(x)}\,dx$$

Subst $u = \ln(\sin(x))$

$$\frac{du}{dx} = \frac{1}{\sin(x)} \cdot \cos(x)$$

$$dx = \frac{du \cdot \sin(x)}{\cos(x)}$$

$x_1 = \frac{\pi}{6} \Rightarrow \ln\left(\sin\frac{\pi}{6}\right)$

$\Rightarrow u_1 = \ln(0{,}5)$

$x_2 = \frac{\pi}{2} = u_2 = \ln(1) = 0$

$$\Rightarrow \int_{\ln 0{,}5}^{0} \frac{u \cdot \overbrace{\cancel{\frac{\sin(x)}{\cos(x)}}}^{\tan(x)}}{\cancel{\tan(x)}}\,du \Rightarrow \int_{\ln(0{,}5)}^{0} u\,du \rightarrow \frac{1}{2}u^2\Big|_{\ln(0{,}5)}^{0}$$

mit $\frac{\sin(x)}{\cos(x)} = \tan(x)$

$$= 0 - \frac{1}{2}(\ln(0{,}5))^2$$

$$\approx -0{,}24$$

$$\int_0^{\frac{\pi}{6}} (2+\cos(x)) \cdot e^{(2x+\sin(x))}\,dx = e^{2x+\sin(x)}\Big]_0^{\frac{\pi}{6}}$$

erkennen $2+\cos(x)$ ist Ableitung des Exponenten $2x+\sin(x)$

$$\Longrightarrow e^{2\cdot\frac{\pi}{6}+\sin\left(\frac{\pi}{6}\right)} - e^{0+0} = e^{\frac{\pi}{3}+0{,}5} - 1 \approx \underline{\underline{3{,}7}}$$

$$\lim_{x\to\frac{\pi}{4}} e^{\frac{4x-\pi}{\cot(\pi-2x)}}$$

$$\boxed{\cot(x) = \frac{1}{\tan(x)}}$$

Betrachte Exponenten

$$\frac{4x-\pi}{\frac{1}{\tan(\pi-2x)}} = \frac{4x-\pi}{\frac{1}{\frac{\sin(\pi-2x)}{\cos(\pi-2x)}}}$$

$$= \frac{4x-\pi}{\underbrace{\frac{\cos(\pi-2x)}{\sin(\pi-2x)}}_{\text{Q.R.}}} \xrightarrow{H.} \frac{4}{\frac{-\sin(\pi-2x)\cdot(-2)\sin(\pi-2x)-\cos(\pi-2x)\cdot\cos(\pi-2x)\cdot(-2)}{\sin^2(\pi-2x)}}$$

$=1 \qquad 2\cdot\frac{\pi}{4}$

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$$

$$= \frac{4\cdot\sin^2(\pi-2x)}{2\cdot\underbrace{\left(\sin^2(\pi-2x)+\cos^2(\pi-2x)\right)}_{=1}} = \frac{4}{2} = \underline{\underline{2}}$$

Grenzwert Exponent

$$\Rightarrow \lim_{x\to\pi/4} = \underline{\underline{e^2}}$$