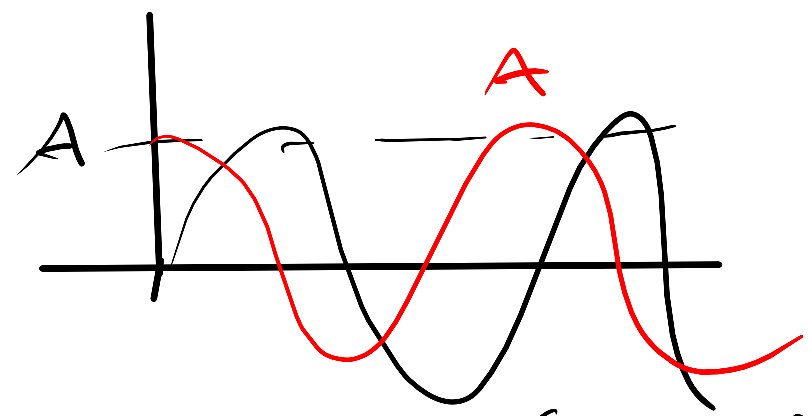
BMC Math. 2 24.6.26

Bestimmen Sie die Überlagerung von

Amplitude $\omega = 3$ Phi

$$1\cdot\cos\left(3t-\frac{\pi}{3}\right)+\sqrt{3}\cdot\sin\left(3t+\frac{5\pi}{3}\right)=A\cdot\sin\left(3t+\phi\right)$$

$A=1$ $A=\sqrt{3}$



$\Rightarrow$ cos in sin verwandeln

$$\cos(\alpha) = \sin\left(\alpha + \overset{\left(\frac{\pi}{2}\right)}{90^\circ}\right)$$

$$=\overset{A_1}{1}\cdot\sin\left(3t\overbrace{-\frac{\pi}{3}+\frac{\pi}{2}}^{\phi_1}\right)+\overset{A_2}{\sqrt{3}}\cdot\sin\left(3t+\overset{\phi_2}{\frac{5\pi}{3}}\right)=A\cdot\sin\left(3t+\phi\right)$$

Tipp: $\arctan\sqrt{3} = \frac{\pi}{3}$ $\arctan\frac{1}{\sqrt{3}} = \frac{\pi}{6}$ $\sqrt{3} = 1{,}73\ldots$

$$A = \sqrt{A_1^2 + A_2^2 + 2 \cdot A_1 A_2 \cos(\phi_1 - \phi_2)}$$

$$\tan\phi = \frac{A_1 \cdot \sin(\phi_1) + A_2 \cdot \sin(\phi_2)}{A_1 \cdot \cos(\phi_1) + A_2 \cdot \cos(\phi_2)}$$

} Formeln

$A_1 = 1 \quad A_2 = \sqrt{3}$

$\phi_1 = \frac{\pi}{6} \quad \phi_2 = \frac{5\pi}{3}$

$\Rightarrow A^2 = 1 + 3 + 2\sqrt{3} \cdot \cos\left(\frac{5\pi}{3} - \frac{\pi}{6}\right) = 4$

$\underline{A = 2}$

$$\tan\phi = \frac{1 \cdot \sin\left(\frac{\pi}{6}\right) + \sqrt{3}\sin\left(\frac{5\pi}{3}\right)}{1 \cdot \cos\left(\frac{\pi}{6}\right) + \sqrt{3} \cdot \cos\left(\frac{5\pi}{3}\right)} = \frac{\frac{1}{2} + \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}\sqrt{3} + \sqrt{3} \cdot \frac{1}{2}} = \frac{-1}{\sqrt{3}} \quad (\tan\phi)$$

$$\phi = \arctan\left(-\frac{1}{\sqrt{3}}\right) = -\arctan\left(\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$$

Ergebnis: $\underline{y(t) = 2 \cdot \sin\left(3t - \frac{\pi}{6}\right)}$

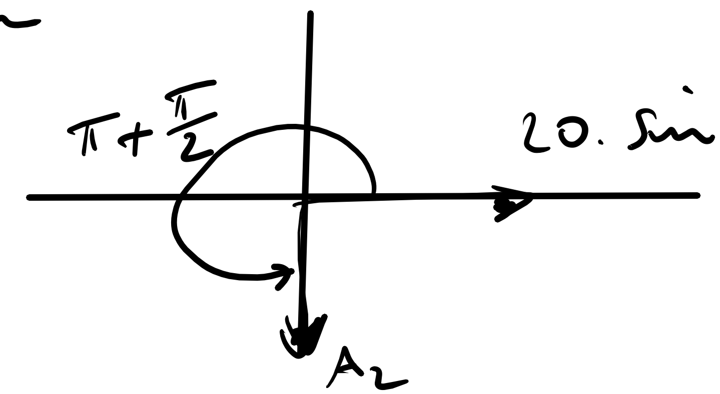
$$\overset{A_1}{10} \cdot \sin\left(x + \frac{\pi}{6}\right) - \overset{A_2}{20} \cos(x) = A \cdot \sin(x + \varphi)$$

$$y_1 = A_1 \cdot \sin(\omega \cdot t + \varphi_1) \qquad y_2 = A_2 \cdot \sin(\omega t + \varphi_2)$$

$$A = \sqrt{A_1^2 + A_2^2 + 2 A_1 A_2 \cos(\varphi_1 - \varphi_2)} \qquad \tan\varphi = \text{- - - — -}$$

$y_2 = -20 \cos(x)$ soll Sinus werden

$$\Rightarrow y_2 = 20 \cdot \sin\left(x + \underbrace{\tfrac{3}{2}\pi}_{\varphi_2}\right)$$



$$A = \sqrt{10^2 + 20^2 + 10 \cdot 10 \cdot \cos(30^\circ - 270^\circ)}$$
$$= 17{,}3$$

$$\tan(\varphi) = \frac{10 \cdot \sin(30) + 20 \cdot \sin(270)}{10 \cdot \cos(30) + 20 \cdot \cos(270)}$$
$$= -1{,}732$$
$$\varphi = -60^\circ$$
$$\triangleq 300^\circ$$

$$u_1(t) = \underbrace{2}_{A_1} \cdot \sin\left(\omega \cdot t + \underbrace{\frac{\pi}{4}}_{\varphi_1}\right)$$

$$u_2(t) = \underline{4 \cdot \cos}\left(\omega t + \frac{\pi}{4}\right)$$

$u_1 + u_2 = u$

$$\left.\begin{matrix} u_1 \\ u_2 \end{matrix}\right\} A \cdot \sin(\omega \cdot t + \varphi)$$

$$\Rightarrow u_2 = 4 \cdot \sin\left(\omega \cdot t + \frac{\pi}{4} + \frac{\pi}{2}\right) = \underbrace{4}_{A_2} \sin\left(\omega \cdot t + \underbrace{\frac{3\pi}{4}}_{\varphi_2}\right)$$

$$A = \sqrt{2^2 + 4^2 + 2 \cdot 2 \cdot 4 \cdot \cos\left(\frac{\pi}{4} - \frac{3\pi}{4}\right)} = 4{,}47$$

$$\tan\varphi = \frac{2 \cdot \sin\left(\frac{\pi}{4}\right) + 4 \cdot \sin\left(\frac{3\pi}{4}\right)}{2 \cdot \cos\left(\frac{\pi}{4}\right) + 4 \cdot \cos\left(\frac{3\pi}{4}\right)} = -1{,}25 \;\hat{=}\; \varphi = -71{,}56^\circ$$

Berechnen Sie Komplex

$$2 \cdot \sin(3t) + \underbrace{3 \cdot \cos(3t + \pi)}_{3 \sin\left(3t + \frac{3}{2}\pi\right)} + \sin\left(3t + \frac{\pi}{2}\right) = A \cdot \sin(3t + \varphi)$$

$$z_1 = 2 \cdot e^{i3t} \cdot e^{0}$$
$$z_2 = 3 \cdot e^{i3t} \cdot e^{i\frac{3\pi}{2}}$$
$$z_3 = 1 \cdot e^{i3t} \cdot e^{i\frac{\pi}{2}}$$

$\Big\}\; t = 0$!

$e^{i(\omega t + \varphi)}$

$t = 0$

$\hat{=}\; e^{i(\cancel{3t} + \varphi)}$

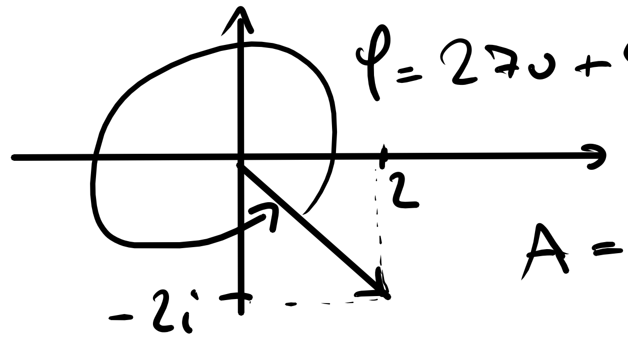
$= \cancel{e^{i3t}} \cdot e^{i\varphi}$

$e^0 = 1 \quad \Rightarrow \quad z_1 = 2$

$$z_2 = 3\Big(\underbrace{\cos \tfrac{3\pi}{2}}_{0} + i \cdot \underbrace{\sin \tfrac{3\pi}{2}}_{-1}\Big) = -3i$$

$$z_3 = 1 \cdot \Big(\underbrace{\cos \tfrac{\pi}{2}}_{0} + i \underbrace{\sin \tfrac{\pi}{2}}_{1}\Big) = i$$

$$\sum_{1}^{3} z_i = 2 - 3i + i = 2 - 2i$$



$\varphi = 270 + 45 = 315^\circ$ oder -45°

$A = \sqrt{2^2 + 2^2} = \sqrt{8}$

$= 2 \cdot \sqrt{2}$

$$2 \cdot \sin\left(2x + \frac{\pi}{3}\right) + 3 \cdot \cos\left(2x - \frac{\pi}{4}\right) = A \cdot \sin(2x + \phi)$$

<u>Ergebnis:</u> $A = 4{,}959$ $\tan\phi = 1{,}235$ $\Rightarrow \phi =$