BMC Math 8.7.26

Berechnen Sie mit Hilfe einer geeigneten Substitution:

$x_2 = \frac{\pi}{2}$

$$\int_{x_1 = 0}^{\frac{\pi}{2}} \frac{\sin(x)}{\sqrt{\cos(x)}}\, dx$$

$u = \cos(x) \Longrightarrow \frac{du}{dx} = -\sin(x)$

$\Rightarrow dx = \frac{du}{-\sin(x)}$

$\underline{u_1} = \cos(x_1) \underline{= 1}$

$\underline{u_2} = \cos(x_2) \underline{= 0}$

$$\Longrightarrow \int_1^0 \frac{\cancel{\sin(x)}}{\sqrt{u}} \cdot \frac{du}{-\cancel{\sin(x)}} = -\int_1^0 \frac{du}{\sqrt{u}} = -\int_1^0 u^{-\frac{1}{2}}\, du = -\left[2u^{\frac{1}{2}}\right]_1^0$$

$$-2 \cdot [0 - 1] = \underline{\underline{2}}$$

Berechnen Sie den Grenzwert $\lim\limits_{x \to 0} \dfrac{\cos(x^2) - \cosh(x^2)}{x^4}$

1) Probieren $\dfrac{\overset{1}{\cos}(0) - \overset{1}{\cosh}(0)}{0^4} = \dfrac{0}{0}$ ↯

2) Hospital $\dfrac{-\sin(x^2)\cdot 2x - \sinh(x^2)\cdot 2x}{4x^3} = \dfrac{-\sin(x^2)\cdot 2 - \sinh(x^2)\cdot 2}{4x^2} \xrightarrow{\text{probieren}}$

$\dfrac{0}{0}$ ↯ nochmal Hospital $\dfrac{-\cos(x^2)\cdot 2\cancel{x}\cdot 2 - \cosh(x^2)\cdot 2\cancel{x}\cdot 2}{8\cancel{x}}$

$= \dfrac{-\cos(x^2)\cdot 4 - \cosh(x^2)\cdot 4}{8} \underset{x \to 0}{=} \dfrac{-4-4}{8} = \underline{\underline{-1}}$

Bestimmen Sie für $a>0$ den Grenzwert:

$$\lim_{x\to\pi} \frac{a\cdot\sin^3(x)}{(x-\pi)-\sinh(x-\pi)} \xrightarrow{H.\,I} \frac{3\cdot a\cdot\left(\sin^2(x)\cdot\cos(x)\right)}{1-\cosh(x-\pi)} = \frac{0}{0} \lightning$$

$$\xrightarrow{H\,II} \frac{3a\cdot\left(2\cdot\sin(x)\cdot\cos(x)\cdot\cos(x) + \sin^2(x)\cdot(-\sin(x))\right)}{-\sinh(x-\pi)} \xrightarrow{H\,III}$$

$$\frac{3a\left(2\cdot\sin(x)\cdot\cos^2(x) - \sin^3(x)\right)}{-\sinh(x-\pi)} \xrightarrow[x\to\pi]{H\,III} \frac{3a\left(2\cdot\cos(x)\cdot\cos^2(x) + \cancel{2\cdot\sin(x)\cdot 2\cdot\cos(x)(-\sin(x))}\ \cancel{-3\cdot\sin^2(x)\cdot\cos(x)}\right)}{\underbrace{-\cosh(x-\pi)}_{-1}}$$

$$= \frac{3a\cdot 2\cdot\cos^3(x)}{-1} \qquad \cos(\pi) = -1 \qquad \Longrightarrow \frac{-6a}{-1} = \underline{\underline{6a}}$$

$$\lim_{x\to\infty} x \cdot \ln\left(\frac{2a+x}{a+x}\right) \qquad a>0$$

für Hôpital: $\dfrac{\ln\left(\frac{2a+x}{a+x}\right)}{x^{-1}}$

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$$

Q.R.

$$\frac{\frac{1}{\left(\frac{2a+x}{a+x}\right)}\cdot\left(\frac{a+x-(2a+x)}{(a+x)^2}\right)}{-\frac{1}{x^2}}$$

$$= \frac{-a}{\left(\frac{2a+x}{\cancel{a+x}}\right)\left(-\frac{1}{x^2}\right)\cdot(a+x)^{\cancel{2}}} = \frac{a\cdot x^2}{2a^2+\underbrace{2ax+ax}+x^2}$$

$$= \frac{ax^2}{2a^2+3ax+x^2} = \frac{a\cancel{x^2}}{\cancel{x^2}\left(\frac{2a^2}{x^2}+\frac{3a}{x}+1\right)} = \frac{a}{\underbrace{\frac{2a^2}{x^2}}_{\to 0}+\underbrace{\frac{3a}{x}}_{\to 0}+1} \; ; \; x\to\infty$$

$$= a$$

$$\lim_{x\to 0} \frac{x\cdot \sin(x)}{\ln\left(x^2+1\right)} \xrightarrow{H} \frac{1\cdot \sin(x) + x\cdot \cos(x)}{\frac{1}{x^2+1}\cdot 2x} \xrightarrow{H}$$

Q. R. f. $\frac{2x}{x^2+1}$

$$= \frac{\cos(x) + \left(\cos(x) - x\cdot \sin(x)\right)}{\frac{2\cdot\left(x^2+1\right) - 2x\cdot 2x}{\left(x^2+1\right)^2}} = \frac{\left(2\cdot\cos(x) - x\cdot\sin(x)\right)\cdot\left(x^2+1\right)^2}{2x^2+2-4x^2}$$

$x \to 0$

$$\frac{\left(2\cdot\cos(0) - \cancel{0\cdot\sin(0)}\right)\left(\cancel{0}+1\right)^2}{0+2-0} = \frac{2}{2} = \underline{\underline{1}}$$