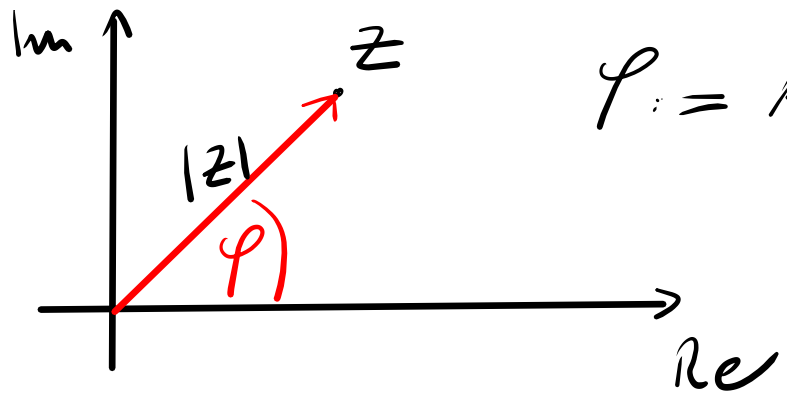
Tutorium Math 1 für B/MC 30.10.26

Komplexe Zahlen (und Winkel zur Re-Achse)



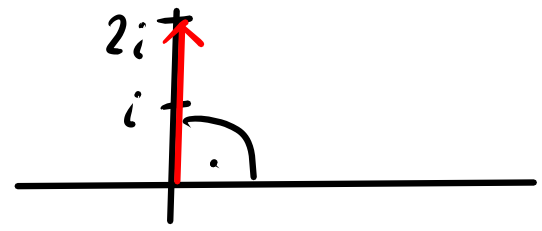
φ := Phi spricht „Fie" Φ := großes Phi

Sei $z = \dfrac{-2+2i}{1+i}$, geben Sie Realteil, Imaginärteil, Betrag und Winkel des Betrages zur Re-Achse an.

$$\Rightarrow \frac{(-2+2i)}{(1+i)} \cdot \frac{(1-i)}{(1-i)} = \frac{-2+2i+2i+2}{2} = \frac{4i}{2} = 2i = z$$

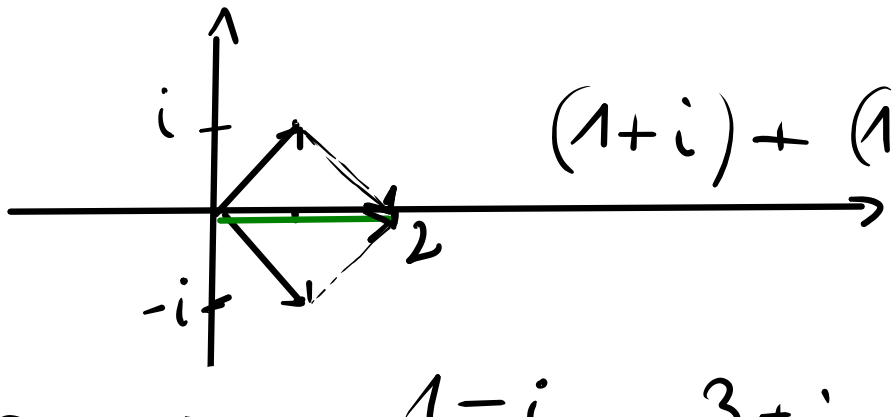
grad = gon
Neugrad (400 gon)

$\Rightarrow Re = 0 \qquad Im = 2 \qquad |z| = 2$



$\varphi = 90° = \frac{\pi}{2}$ (Rad)

oder $|z| = \sqrt{(0+2i)(0-2i)} = \sqrt{4i^2} = 2i$ q.e.d



$(1+i) + (1-i) = 2$

Sei $z = -\frac{1-i}{2+i} - \frac{3+i}{4}$

$$\Rightarrow \quad \frac{-1+i}{2+i} + \frac{-3-i}{4} \quad \Rightarrow \quad \frac{(-1+i)(2-i)}{(2+i)(2-i)} + \frac{-3-i}{4}$$

$$(2+i)(2-i) = 4 - 2i + 2i + 1 = 5$$

$$\Rightarrow \quad \frac{-2+i+2i-i^2}{5} + \frac{-3-i}{4}$$

$$(-2-i)(-2+i) \quad 2-i-4$$

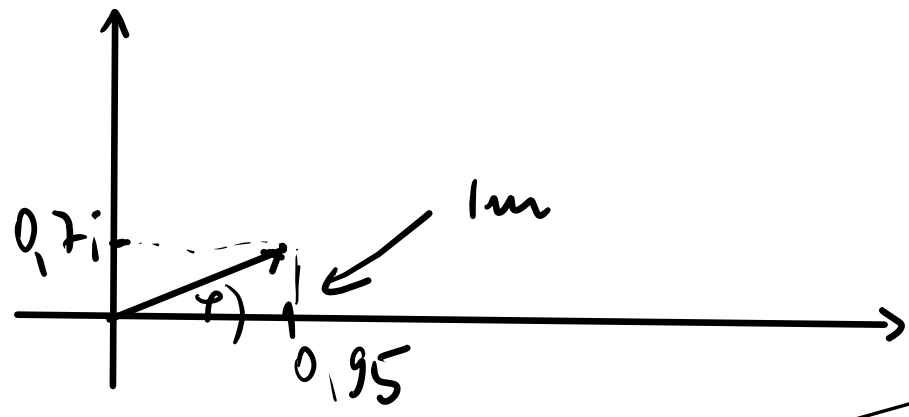
$$\Rightarrow \quad \frac{-1+3i}{5} + \frac{-3-i}{4} = \left(-\frac{1}{5}\right) + \left(\frac{3}{5}\right)i \left(-\frac{3}{4}\right) - \left(\frac{1}{4}\right)i = \frac{-4-15}{20} + \frac{12-5}{20}\,i$$

$$\mathrm{Re} \quad -\frac{19}{20} + \frac{7}{20}\,i \qquad \mathrm{Im}$$

$$|z| = \sqrt{\left(-\frac{19}{20} + \frac{7}{20}i\right)\left(-\frac{19}{20} - \frac{7}{20}i\right)}$$

$$|z| = \sqrt{\frac{361}{400} + \frac{49}{400}} = \sqrt{\frac{41}{40}} \simeq 1$$

φ:



$z = \frac{19}{20} + \frac{7}{20}i$

$\frac{7}{20} \cdot \frac{20}{19}$

$\tan\varphi = \frac{\frac{7}{20}}{\frac{19}{20}} \approx 0{,}368$

$\Rightarrow \varphi = \arctan 0{,}368$

$\varphi \approx 20{,}22°$

Esels brücke

Winkel°	0°	30°	45°	60°	90°
Sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
Cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$

$$z = \frac{(1+i)^2}{2} - \frac{6+5i}{i^3} \qquad i^3 = i^2 \cdot i$$

$$= \frac{1 + 2i + i^2}{2} - \frac{6+5i}{-i} = i - \frac{(6+5i)\cdot i}{-i \cdot i}$$

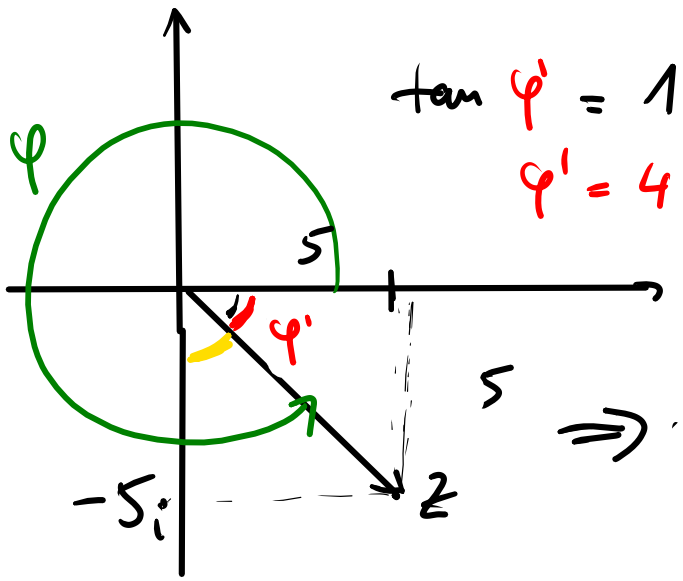
$$= i - \frac{6i - 5}{1} = i - 6i + 5 = \underline{\underline{5 - 5i}}$$

$$i^{121} = i \cdot i^{120}$$

$$i \cdot \left(i^2\right)^{60}$$

$$i \cdot (-1)^{60} \qquad +1$$

$$\Rightarrow \underline{\underline{i}}$$



$$\tan \varphi' = 1 = \frac{5}{5}$$

$$\varphi' = 45^\circ$$

180
90
45

315°

$$|z| = \sqrt{25 + 25} = \sqrt{50} \approx \underline{\underline{7{,}07}}$$

$$|z|^2 = (5 - 5i)(5 + 5i) = 25 + 25i - 25i + 25$$

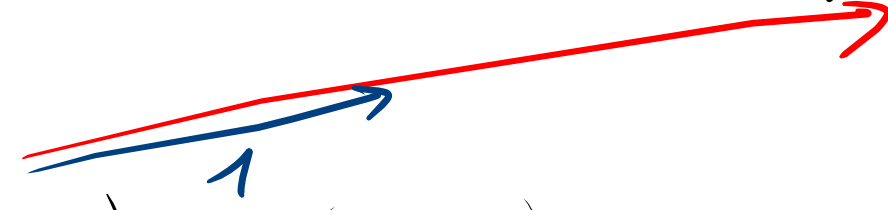
$$\Rightarrow \varphi = 360^\circ - 45^\circ = \underline{\underline{315^\circ}}$$

Sei $\vec{v} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ Verwandeln sie diesen Vektor in einen der den Betrag 1 hat (Einheitsvektor)

$|\vec{v}| = \sqrt{1^2 + 3^2 + (-2)^2}$

$= \sqrt{1 + 9 + 4} = \sqrt{14}$

$\vec{v}_e = \frac{\vec{v}}{|\vec{v}|}$



$= \begin{pmatrix} \frac{1}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \\ \frac{-2}{\sqrt{14}} \end{pmatrix} \Rightarrow$ Betrag 1

$\frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 0,267 \\ 0,802 \\ -0,534 \end{pmatrix}$

$\Rightarrow \sqrt{0,267^2 + 0,802^2 + (-0,534)^2} \approx 1$