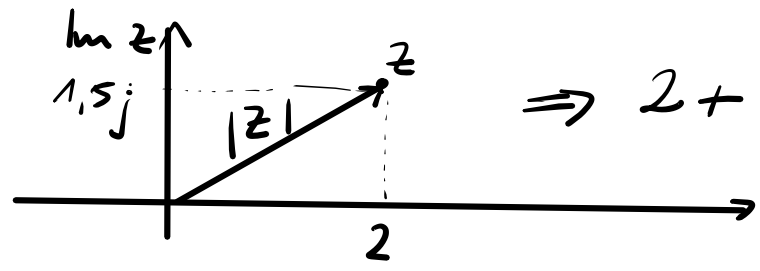
Tutorium M1 f. BMC 23.10.26

Komplexe Zahlen



$\Rightarrow 2 + 1{,}5j$ oder $2 + 1{,}5i$

Übung: $z_1 = 2 + 3j$, $z_2 = 2 - j$, $z_3 = 2j + 1$ gegeben

Re z

oder $z \cdot z^*$

$|z|^2 = z \cdot \bar{z}$ es ist

1) $\frac{z_1 + z_2}{z_3} = \frac{2 + 3j + 2 - j}{1 + 2j} = \frac{4 + 2j}{1 + 2j}$

Re und Im angeben

$\Rightarrow \frac{4 - 8j + 2j + 4}{1 - 2j + 2j + 4} = \frac{8 - 6j}{5}$

$= \frac{8}{5} - \frac{6}{5}j \Rightarrow Re\, z = \frac{8}{5}$ $Im\, z = -\frac{6}{5}$

Nenner reell machen

$\frac{(4 + 2j) \cdot (1 - 2j)}{(1 + 2j) \cdot (1 - 2j)}$

$1^2 + 2^2 = 5$

2) $4 z_2 \cdot z_3 = 4 \cdot (2-j)(1+2j) = 4 \cdot (2 + 4j - j \underbrace{-2j^2}_{+2}) \qquad j^2$

$= 4(4 + 3j) = \underbrace{16}_{\text{Re } z} + \underbrace{12}_{\text{Im } z} j$

3) $\frac{z_1}{z_2^*} = \frac{2+3j}{2+j} = \frac{(2+3j)(2-j)}{\underbrace{(2+j)(2-j)}_{5}}$

$= \frac{4 - 2j + 6j + 3}{5}$

$= \frac{7 + 4j}{5}$

$= \underbrace{\frac{7}{5}}_{\text{Re } z} + \underbrace{\frac{4}{5}}_{\text{Im } z} j$

$(2+j)(2-j) = 5$ weil: $4 - \cancel{2j} + \cancel{2j} - j^2 = 4 - (-1) = 4 + 1 = 5$

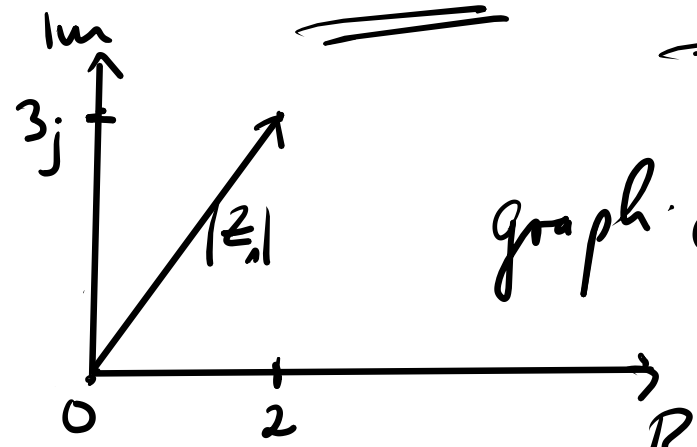
wenn $z = (a + jb)$

$\Rightarrow \bar{z}$ oder $z^+ = (a - jb)$

$|z_1| = \sqrt{z_1 \cdot z_1^*}$ weil $z_1 \cdot z_1^* = |z_1|^2$

$\Rightarrow \sqrt{(2+3j)(2-3j)} = \sqrt{4 - 6j + 6j + 9} = \underline{\underline{\sqrt{13}}} \simeq \underline{\underline{3,6}}$

5) $\left|\frac{z_2}{z_3}\right| = \left|\frac{2-j}{1+2j}\right|$



graph. Lösung

$= \left|\frac{(2-j)(1-2j)}{(1+2j)(1-2j)}\right| = \left|\frac{2-4j-j-2}{5}\right| = \left|\frac{-5j}{5}\right| = \underline{|-j|} = \underline{\underline{1}}$

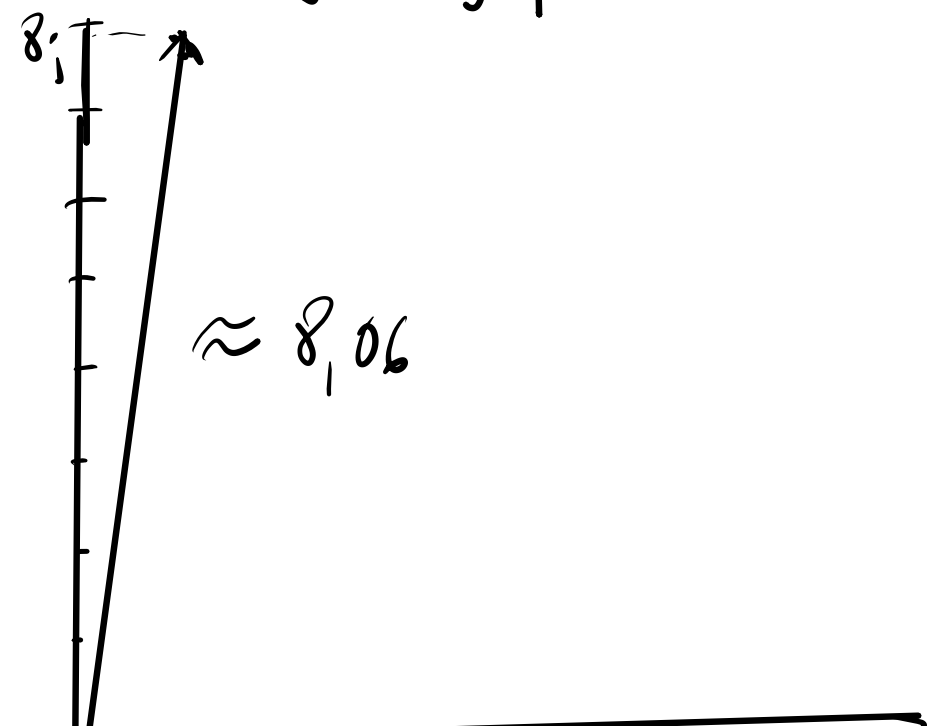
$\operatorname{Re} z = 0 \quad \operatorname{Im} z = -1$ aber $|z|$

$$|z_1 \cdot z_2^*| = |(2+3j)(2+j)| = |4+2j+6j+3j^2|$$

$$= |4+8j-3| = |1+8j| = \sqrt{65}$$

$$\Rightarrow \sqrt{(1+8j)(1-8j)} = \sqrt{1-8j+8j+64}$$

$$= \sqrt{65}$$



Thema: Winkel zwischen 2 Vektoren

Über Skalarprodukt:

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos(\alpha)$$

$$\cos(\alpha) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\vec{a} \cdot \vec{b} = \sum_{1}^{3} a_i \cdot b_i$$

Winkel zwischen $\vec{a} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$ und $\vec{b} \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$

$$|\vec{a}| = \sqrt{(-1)^2 + 0^2 + 3^2} = \sqrt{10}$$

$$|\vec{b}| = \sqrt{1^2 + 3^2 + (-4)^2} = \sqrt{26}$$

$$\vec{a} \cdot \vec{b} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} = (-1 \cdot 1) + (0 \cdot 3) + (3 \cdot (-4)) = \underline{-13}$$

$$= \cos(\alpha) = \frac{-13}{\sqrt{10} \cdot \sqrt{26}} = -0{,}806 \Rightarrow \alpha = \underline{143{,}72^\circ}$$